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UNITED KINGDOM DEBT MANAGEMENT OFFICE

Issuance of ultra-long gilt instruments Consultation Document

2 December 2004

DMO consultation on ultra-long and annuity gilts

Introduction

1. In today's Pre-Budget Report 2004 the Chancellor of the Exchequer has asked the UK Debt Management Office (DMO) to consult market participants on the possible introduction of additional gilt instruments, specifically:

- ultra-long (circa 50-year) conventional and index-linked gilts; and
- ultra-long (circa 50-year) conventional and index-linked annuity-type gilts.

2. Consequently, the DMO has issued this consultation paper, which seeks views from any interested parties on the possible introduction of such new gilts. A list of questions on which the DMO is seeking feedback is in paragraph 22.

3. Feedback, in written format[1] should be provided to the DMO by Friday 21 January 2005. Written responses should be sent to Steve Whiting at the DMO, Eastcheap Court, 11 Philpot Lane, London EC3M 8UD. All interested parties are invited to respond. The DMO would of course be ready to meet any respondent bilaterally or collectively in addition to the provision of written feedback.

4. On the basis of the feedback received the DMO will make recommendations to Treasury Ministers about whether to issue a new format gilt in 2005-06. Any plans to do so would be included in the DMO remit for 2005-06.

5. Please contact Arnaud Marès (tel 020 7862 6615 / arnaud.mares@dmo.gsi.gov.uk) or Steve Whiting (tel 020 7862 6532 / steve.whiting@dmo.gsi.gov.uk) if you have any questions on the consultation or would like to arrange a meeting.

Background

6. HM Government's debt management policy objective is:

> "*to minimise over the long term, the costs of meeting the Government's financing needs, taking into account risk, whilst ensuring that debt management policy is consistent with the aims of monetary policy*".

7. The debt management objective is achieved in particular by pursuing an open, transparent and predictable issuance policy, developing a liquid and efficient

[1] You should note that the DMO and HM Treasury are subject to the provisions of the Freedom of Information Act 2000 and consequently information disclosed by us in response to requests for information under the Act could enter the public domain. If you are providing information that is commercially sensitive please mark it as such and we will endeavour not to disclose it to the extent that such non-disclosure is permissible under the Act.

gilts market and adjusting the maturity and nature of the Government's debt portfolio.

8. In order to determine the maturity and composition of debt issuance, the Government takes into account, in addition to its own appetite for risk, the shape of the yield curves (both nominal and real), the expected effects of issuance policy and investors' demand for gilts.

9. Reflecting these influences, HM Government's debt issuance is, relative to other major governments, already skewed towards longer maturity bonds. At end-September 2004, long-dated gilts (with maturities of 15 years or longer) accounted for 31% of the nominal gilt portfolio. The longest nominal gilt currently in issue is 4¾% Treasury Stock 2038 and the longest index-linked gilt currently in issue is 2% Index-linked Treasury Stock 2035.

10. The largest group of investors in gilts is the domestic pension industry. According to National Statistics data, UK pension funds and insurance companies hold around 64% of all gilts and an even larger proportion of index-linked and long maturity gilts.

11. As part of its continuous dialogue with investors, the DMO has already held informal talks this summer with several pension industry participants in order to gauge the size and nature of the industry's demand for gilts going forward. These talks suggested that demand for high-quality bonds from the UK pension industry and other investors is likely to increase in the future. There are likely to be several factors at work here; not only demographic trends but also a possible change in risk management practices (i.e. closer matching of assets and liabilities, reflected in a shift from equities to bonds in pension portfolios) and the likelihood that a shift from Defined Benefit to Defined Contribution schemes will increase demand for annuities.

12. Excess demand for high quality inflation-linked bonds and very long-dated bonds in the formats desired by long-term investors has featured strongly in the DMO's informal discussions. It seems likely that both HM Government and investors may benefit from the issuance of bonds in maturity ranges or formats where there currently exists either no or insufficient supply.

13. In particular, it has been suggested that HM Government should consider issuing:

- gilts with maturities longer than currently exist, circa 50 years, in conventional or index-linked format;

- bonds with an alternative cash flow structure – fixed-term annuities – in conventional or index-linked format;

- index-linked bonds with limited price indexation properties (LPI bonds), i.e. where the indexation of the cash flows to the reference price index is capped to the upside and/or to the downside; and

- so-called ‘longevity’ bonds, i.e. bonds whose return is contingent on a reference population’s longevity.

14. These issues are pertinent to the general debate on pensions and increased longevity. The Pensions Commission has provided a valuable analysis of the long-term challenges posed by an ageing population and the changing patterns in the nature of private pension provision[2]. The Commission is also currently consulting on longevity risk issues generally.

15. Given the ongoing policy debate on ageing, the issuance of ‘longevity’ bonds is not envisaged in 2005-06. Such instruments would raise broader policy issues – such as the outright transfer of additional longevity risk onto the Government’s balance sheet – that extend beyond a strict interpretation of debt management considerations. However, the DMO and HM Treasury may revisit this issue at a later stage and are interested in the market’s views on the demand for such instruments.

16. As underlined earlier, when pursuing its debt management objective to minimise cost, subject to risk, HM Government is concerned with ensuring that the gilts market remains liquid and efficient. HM Government aims at issuing gilts that achieve a benchmark premium and are potentially attractive to a broad investor base.

17. HM Government is therefore not inclined to issue instruments that are likely to appeal to a limited group of investors or that may lead to a fragmentation of the gilts market, with a resulting loss of liquidity. It is, therefore, not inclined to pursue the option of issuing LPI bonds.

18. There are no particular prior reservations to HM Government issuing gilts with ultra-long maturities or in annuity format. A decision to do so would be based on such issuance delivering cost and / or risk benefits, consistent with HM Government’s overall debt management policy objective.

New index-linked gilt design

19. The DMO is also taking the opportunity to introduce a new indexation structure into any new index-linked gilts issued from 2005-06. Specifically, any new index-linked gilt issued from 2005-06 will use a three as opposed to the current eight-month indexation lag to the RPI. This reflects the adoption of a three-month lag as current international best practice. Attached for information in Annex A is the indexation methodology and relevant formulae. Note that the DMO does not intend to introduce a deflation floor. The introduction of any new three-month lag index-linked gilts from 2005-06 does not mean that current eight-month lag index-linked gilts cannot be re-opened.

[2] http://www.pensionscommission.org.uk/publications/2004/annrep/fullreport.pdf

Scope of the consultation

20. This DMO consultation is focused on potential issuance in 2005-06 of ultra-long gilts and annuity gilts. As noted above, the Pensions Commission is consulting on pensions and longevity issues generally, and the DMO will in due course be interested to see responses which may have a future impact on the gilts market.

21. The DMO acknowledges that the issuance of an annuity-type gilt, i.e. a gilt where all cash flows are constant in either real or nominal terms over the life of the bond, would be a significant innovation for the gilts market. For the sake of clarity, a description of the form that such instruments might take is provided in Annex B. Comments are invited on the proposed format.

22. The DMO is seeking feedback on the following questions:

A. Potential demand for new instruments in 2005-06

a) What is the potential scale of demand for ultra-long (circa 50-year) conventional or index-linked gilts?

b) If the DMO were to extend the maturity range of gilts it issues, for which maturities would there be most potential demand?

c) What is the potential demand for gilts structured in annuity format?

d) How sustainable would demand be for ultra-long and annuity format gilts over time?

e) In the event that it was decided to issue only a single form of new instrument in 2005-06, which of the instruments outlined above is of **most** potential interest?

f) How much, in £ billions should the Government initially plan to supply in one financial year?

B. Issuance procedure

g) If the DMO were to issue any such new instruments, should issuance take place from the outset through auctions?

h) Alternatively, should the DMO consider other means to distribute the bonds when first issued (or perhaps on an ongoing basis) and if so, which?

C. Instrument design

i) If the DMO were to issue ultra-long and/or annuity gilts, should such issuance take place preferably in conventional format or in index-linked format?

j) If the DMO were to issue gilts with an annuity structure (in nominal or index-linked format), should the DMO allow for these new gilts to be stripped?

k) For new index-linked gilts with a three-month lag, and/or for any new gilts issued in annuity format, the DMO would be interested in views on whether such gilts should be held only in CREST. This would allow a zero ex-dividend period to be introduced for those gilts. Respondents may wish to take account of possible liquidity issues arising from the need to make payments into Cash Memorandum Accounts (CMAs).

l) If the DMO were to issue gilts in annuity format, what are stakeholders' preferences regarding the frequency of cash flows?

m) Do recipients have any other comments or suggestions on the proposed product design in Annex B?

D. Timing for introduction of new instruments

n) If the DMO were to issue any such instruments – and gilt annuities in particular – what would be the lead times required by investors, Gilt-edged Market Makers (GEMMs) and other interested stakeholders before such issuance can realistically take place?

o) What are the lead times required by investors, GEMMs and other interested stakeholders before issuance of a three-month lag index-linked gilt can take place?

E. Potential advantages / disadvantages with issuance of new instruments

p) How do respondents assess the benefits, for HM Government and for investors, of any new instrument being created?

q) Would the adoption of an annuity format detract from the liquidity and benchmark status of such bonds?

r) Would the introduction of an annuity format detract from the liquidity of standard benchmark gilts (particularly as the annuity format bonds age), or otherwise fragment the overall gilts market?

F. Market maker responsibilities

s) If the DMO were to issue any such instrument, should the roles and responsibilities of GEMMs be identical to those applying to other gilts?

ANNEX A: Method of indexation for index-linked gilts with a three-month lag

Shared features with existing index-linked gilts

- Indexation will be to the Retail Prices Index (RPI)[3].
- Coupon payments and the redemption payment will be indexed.
- Coupon payments will be made semi-annually.
- There will be no deflation floor on the principal repayment value.
- The daycount convention to be used for both discounting (i.e. price / yield calculations) and accrued interest calculations will be actual / actual.
- The DMO has no current plans to introduce a strips facility for index-linked gilts but might consider introducing such a facility in the future, subject to consultation.

Indexation methodology

Any new index-linked gilt would employ the three-month lag indexation technique first used in the Canadian Real Return Bond (RRB) market, rather than the eight-month lag methodology used for existing index-linked gilts. In addition to using a shorter lag, RRB indexation is applied in a significantly different way from that for existing index-linked gilts.

An Index Ratio is applied to calculate the coupon payments, the redemption payment (i.e. the uplifted principal) and the accrued interest. The Index Ratio for a given date is defined as the ratio of the reference RPI applicable to that date (“$\text{Ref RPI}_{\text{Date}}$”) divided by the reference RPI applicable to the original issue date of the gilt (“$\text{Ref RPI}_{\text{First Issue Date}}$”), rounded to the nearest 5th decimal place:

$$\text{Index Ratio}_{\text{Date}} = \left[\frac{\text{Ref RPI}_{\text{Date}}}{\text{Ref RPI}_{\text{First Issue Date}}}\right], \text{ rounded to the nearest 5}^{\text{th}} \text{ decimal place.}$$

The reference RPI for the first calendar day of any calendar month is the RPI for the calendar month falling three months earlier. For example, the reference RPI for 1 June

[3] The DMO/HMT currently has no plans to use any price index other than the RPI for new index-linked gilts, but if such a move were to be considered in the future the DMO would consult with market participants and investors at the appropriate time.

corresponds to the RPI for March, the reference RPI for 1 July corresponds to the RPI for April, etc. The reference RPI for any other day in the month is calculated by linear interpolation between the reference RPI applicable to the first calendar day of the month in which the day falls and the reference RPI applicable to the first calendar day of the month immediately following. Interpolated values for $\text{Ref RPI}_{\text{Date}}$ should be rounded to the nearest 5th decimal place, as should values for $\text{Index Ratio}_{\text{Date}}$.

The formula used to calculate $\text{Ref RPI}_{\text{Date}}$ can be expressed as follows:

$$\text{Ref RPI}_{\text{Date}} = \text{Ref RPI}_{\text{M}} + \left(\frac{t-1}{D}\right)\left[\text{Ref RPI}_{\text{M+1}} - \text{Ref RPI}_{\text{M}}\right]$$

where:

D = The number of days in the calendar month in which the given date falls.

t = The calendar day corresponding to the given date.

$\text{Ref RPI}_{\text{M}}$ = Reference RPI for the first day of the calendar month in which the given date falls.

$\text{Ref RPI}_{\text{M+1}}$ = Reference RPI for the first day of the calendar month immediately following the given date.

For example, the reference RPI for 20 July 2001 is calculated as follows:

$$\text{Ref RPI}_{\text{20 July 2001}} = \text{Ref RPI}_{\text{1 July 2001}} + \left(\frac{19}{31}\right)\left[\text{Ref RPI}_{\text{1 August 2001}} - \text{Ref RPI}_{\text{1 July 2001}}\right]$$

$$= \text{RPI}_{\text{April 2001}} + \left(\frac{19}{31}\right)\left[\text{RPI}_{\text{May 2001}} - \text{RPI}_{\text{April 2001}}\right]$$

$$= 173.1 + \left(\frac{19}{31}\right)\left[174.2 - 173.1\right] = 173.77419$$, when rounded to the nearest 5th decimal place.

The $\text{Ref RPI}_{\text{First Issue Date}}$ for a given bond remains constant over its life. However, different index-linked gilts should have different values for $\text{Ref RPI}_{\text{First Issue Date}}$ (depending on when they are first issued).

Calculation of coupon payments

For an index-linked gilt the semi-annual coupon payments per £100 nominal are calculated as the product of the real coupon per £100 nominal and the relevant value of the Index Ratio:

$$\text{Coupon Payment}_{\text{Dividend Date}} = \frac{c}{2} \times \text{Index Ratio}_{\text{Dividend Date}}$$

where: c = (Real) coupon per £100 nominal.

Coupon payments will be rounded to the nearest 6th decimal place per £100 nominal.

Ex-dividend period

For the purpose of this annex, it is assumed that index-linked gilts with a three-month indexation lag would carry the standard ex-dividend period of seven business days. However, views on whether index-linked gilts with a three-month indexation lag should have a zero ex-dividend period by restricting ownership to holdings in CREST are sought as part of this consultation (Question k).

Calculation of the redemption payment

The redemption payment per £100 nominal is calculated as follows:

$$\text{Redemption Payment} = 100 \times \text{Index Ratio}_{\text{Redemption Date}}$$

The redemption payment will be rounded to the nearest 6th decimal place per £100 nominal[4].

[4] Though in practice, for an index-linked gilt with a three-month lag the 4th, 5th and 6th decimal places for the redemption payment will be 0 due to the rounding of the Index Ratio.

Note: unlike in some sovereign index-linked bond markets, in the UK no deflation floor will be applied when calculating the redemption payment i.e. the redemption payment for an index-linked gilt could fall below £100 per £100 nominal if $\text{Ref RPI}_{\text{Redemption Date}}$ were less than $\text{Ref RPI}_{\text{First Issue Date}}$.

When does the redemption payment become known?

To illustrate when the redemption payment (and the final coupon payment) will be fixed, consider some hypothetical cases based on the assumption of an index-linked gilt with a three-month lag redeeming on different dates in December 2003.

Case 1: Redemption on 1 December 2003

The redemption payment would have been fixed when the September 2003 RPI was published on 14 October, i.e. the redemption payment would have been known approximately 6 weeks (48 days) before the bond redeemed.

Case 2: Redemption on 2 December 2003

The redemption payment would have been fixed when the October 2003 RPI was published on 18 November, i.e. the redemption payment would have been known 2 weeks (14 days) before the bond redeemed.

Case 3: Redemption on 31 December 2003

The redemption payment would have been fixed when the October 2003 RPI was published on 18 November, i.e. the redemption payment would have been known approximately 5 weeks (43 days) before the bond redeemed.

So, in practice, the redemption payment and the final dividend payment on an index-linked gilt with a three-month lag will be fixed around 2-6 weeks before the redemption date.

Calculation of the settlement price

Index-linked gilts with a three-month lag will trade and be auctioned on the basis of the Real Clean Price per £100 nominal (real prices will be quoted to 2 decimal places).

The Inflation-Adjusted Clean Price per £100 nominal is calculated from the real clean price using the following formula:

$$\text{Inflation-Adjusted Clean Price} = \text{Real Clean Price} \times \text{Index Ratio}_{\text{Set Date}} \quad \text{(this should be left unrounded)}$$

The Inflation-Adjusted Dirty Price per £100 nominal is calculated as:

$$\begin{aligned}\text{Inflation-Adjusted Dirty Price per £100 nominal} &= \text{Inflation-Adjusted Clean Price per £100 nominal} \\ &\quad + \text{Inflation-Adjusted Accrued Interest per £100 nominal}\end{aligned}$$

(this should be left unrounded)

where: $\text{Inflation-Adjusted Accrued Interest} = \text{Real Accrued Interest} \times \text{Index Ratio}_{\text{Set Date}}$

and the Real Accrued Interest (*RAI*) is defined below. The Inflation-Adjusted Accrued Interest should be left unrounded.

Calculation of redemption yields from real prices

For price / yield calculations, compounding will occur on quasi-coupon dates. Quasi-coupon dates are the dates on the semi-annual cycle defined by the maturity date, irrespective of whether cash flows occur on those dates (examples of quasi-coupon dates on which cash flows would not occur include the first quasi-coupon date of a new issue having a long first dividend period; the next quasi-coupon date of a gilt settling in its ex-dividend period; and most quasi-coupon dates of a strip). A full (quasi-) coupon period is defined as the period between two consecutive quasi-coupon dates. For example, a gilt settling on its issue date (assuming this is not also a quasi-coupon date) will have a quasi-coupon period which starts on the quasi-coupon date prior to the issue date and ends on the first quasi-coupon date following the issue date. Cash flows and quasi-coupon dates which are due to occur on non-business days are not adjusted (i.e. are not ‘bumped’) for price/yield calculations.

The DMO price/yield formula is defined as follows:

(1) For trades settling before the penultimate dividend date:

$$P = w^{\frac{r}{s}}\left[d_1 + d_2 w + \frac{cw^2\left(1 - w^{n-1}\right)}{2\left(1 - w\right)} + 100w^n \right] \quad , \quad n \geq 1$$

In the event that the formula were to be used to derive a yield from a price it is not possible (in most cases) to solve for yield in terms of price algebraically, and so some form of numerical technique will need to be used if, given a price, a value for the redemption yield is required[5].

(2) For trades settling on or after the penultimate dividend date and where the trade date is before the publication date of the RPI that determines the redemption payment[6]:

$$P = w^{\frac{r}{s}}\left(d_1 + 100\right) \quad , \quad n = 0$$

In this case, it is possible to solve algebraically for yield in terms of price:

$$\rho = 2 \times \left[\left(\frac{P}{d_1 + 100} \right)^{-\frac{s}{r}} - 1 \right]$$

(3) Where the trade occurs after the publication of the RPI that determines the redemption payment, the index-linked gilt will effectively become a nominal (rather than

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[5] In order to solve some types of equation it is necessary to obtain numerical approximations to the roots using an iterative process. An iterative process starts with an approximation x_0 to a root λ from which another approximation x_1 is obtained, and then another approximation x_2, and so on. For an effective process (for a particular root) the successive values (or iterates) x_1, x_2, x_3,... should become progressively closer to the root λ . The process is continued until an approximation of the required accuracy is obtained.

[6] Some examples to illustrate which RPI determines the redemption payment are provided earlier in this Annex.

real) instrument and the formula for calculating the (real) dirty price from the nominal yield will be given by:

$$P = \left(\frac{1}{\text{Index Ratio}_{\text{Set Date}}} \right) \left[v^{\frac{r}{s}} \left(D_{LAST} + R \right) \right]$$

In this case, it is possible to solve algebraically for yield in terms of price:

$$y = 2 \times \left[\left(\frac{P \times \text{Index Ratio}_{\text{Set Date}}}{D_{LAST} + R} \right)^{-\frac{s}{r}} - 1 \right]$$

where:

P = Real dirty price per £100 nominal.

c = (Real) coupon per £100 nominal.

r = Number of calendar days from the settlement date to the next quasi-coupon date ($r = s$ if the settlement date occurs on a quasi-coupon date).

r_1 = Number of calendar days from the issue date to the first quasi-coupon date.

s = Number of calendar days in the full quasi-coupon period in which the settlement date occurs (i.e. between the prior quasi-coupon date and the following quasi-coupon date). If the settlement date occurs on a quasi-coupon date, s is measured for the quasi-coupon period starting on the settlement date.

s_1 = Number of calendar days in the full quasi-coupon period in which the issue date occurs.

n = Number of full quasi-coupon periods between the next quasi-coupon date after the settlement date and the redemption date.

ρ = Semi-annually compounded real redemption yield (decimal) i.e. if the real yield is 2.5% then ρ = 0.025.

w = $\dfrac{1}{\left(1 + \dfrac{\rho}{2}\right)}$

y = Semi-annually compounded nominal redemption yield (decimal) i.e. if the nominal yield is 5% then y = 0.05.

v = $\dfrac{1}{\left(1 + \dfrac{y}{2}\right)}$

D_{LAST} = Final (fixed) coupon payment per £100 nominal of the gilt, as published.

R = Redemption payment (fixed) per £100 nominal of the gilt, as published.

The terms d_1 and d_2 represent the <u>real</u> dividend payments (see below). These are left unrounded since they are not the cash flows actually paid to holders of the bonds. Holders will receive the real dividends multiplied by the relevant Index Ratio and then rounded.

(1) In the case of settlement in a <u>standard</u> dividend period:

(a) If the settlement date occurs on or before the ex-dividend date:

$$d_1 = \frac{c}{2}$$

$$d_2 = \frac{c}{2}$$

(b) If the settlement date occurs after the ex-dividend date:

$$d_1 = 0$$

$$d_2 = \frac{c}{2}$$

(2) In the case of settlement in a <u>short</u> first dividend period:

(a) If the settlement date is on or before the ex-dividend date:

$$d_1 = \frac{r_1}{s_1} \times \frac{c}{2}$$

$$d_2 = \frac{c}{2}$$

(b) If the settlement date occurs after the ex-dividend date:

$$d_1 = 0$$

$$d_2 = \frac{c}{2}$$

(3) In the case of settlement in a <u>long</u> first dividend period:

(a) If the settlement date is in the first quasi-coupon period:

$$d_1 = 0$$

$$d_2 = \left(1 + \frac{r_1}{s_1}\right) \times \frac{c}{2}$$

(b) If the settlement date is in the second quasi-coupon period and on or before the ex-dividend date:

$$d_1 = \left(1 + \frac{r_1}{s_1}\right) \times \frac{c}{2}$$

$$d_2 = \frac{c}{2}$$

(c) If the settlement date occurs during the second quasi-coupon period after the ex-dividend date:

$$d_1 = 0$$

$$d_2 = \frac{c}{2}$$

Calculation of the real accrued interest

(1) Standard dividend periods

$$RAI = \begin{cases} \frac{t}{s} \times \frac{c}{2} & \text{if the settlement date occurs on or before the ex - dividend date} \\ \left(\frac{t}{s} - 1\right) \times \frac{c}{2} & \text{if the settlement date occurs after the ex - dividend date} \end{cases}$$

where all the terms are as above, and:

t = Number of calendar days from the previous quasi-coupon date to the settlement date (t = 0 if the settlement date occurs on a quasi-coupon date).

Note: $s = s_1$ for trades settling in the first quasi-coupon period.

(2) Short first dividend periods

$$RAI = \begin{cases} \frac{t^*}{s_1} \times \frac{c}{2} & \text{if the settlement date occurs on or before the ex - dividend date} \\ \left(\frac{t^* - r_1}{s_1}\right) \times \frac{c}{2} & \text{if the settlement date occurs after the ex - dividend date} \end{cases}$$

where all terms are as above, and:

t^* = Number of calendar days from the <u>issue</u> date to the settlement date.

(3) Long first dividend periods

$$RAI = \begin{cases} \frac{t^{**}}{s_1} \times \frac{c}{2} & \text{if the settlement date occurs during the first quasi - coupon period} \\ \left(\frac{r_1}{s_1} + \frac{r_2}{s_2}\right) \times \frac{c}{2} & \text{if the settlement date occurs during the second quasi - coupon period on or before the ex - dividend date} \\ \left(\frac{r_2}{s_2} - 1\right) \times \frac{c}{2} & \text{if the settlement date occurs during the second quasi - coupon period after the ex - dividend date} \end{cases}$$

where all terms are as above, and:

t^{**} = Number of calendar days from the issue date to the settlement date in the first quasi-coupon period (this term only applies if the gilt settles in the first quasi-coupon period).

r_2 = Number of calendar days from the quasi-coupon date after the issue date to the settlement date in the quasi-coupon period in which the issue date occurs (this term only applies if the gilt settles in the second quasi-coupon period).

s_2 = Number of calendar days in the full quasi-coupon period after the quasi-coupon period in which the issue date occurs.

ANNEX B: Design of annuity gilts

This annex sets out a proposal for the design of potential new types of gilt in fixed maturity annuity format (conventional and index-linked annuities). This is provided for indication and comments are invited on any aspect of the proposed structure as set out below, including any accounting or other issues which may arise. In providing comments, readers should take into account that it is in principle the DMO's intention that such instruments would be re-opened once issued, and would encourage the development of a reasonably active secondary market.

Common features

Aspects of the design which would be common to both conventional and index-linked annuities are as follows.

Maturity

It is likely that annuities would have a very long maturity at issuance – circa 50 years. However, any maturity is in principle possible.

Annuity Payment frequency

This annex is based on the assumption that annuities would make a payment twice a year. However, payment frequency on annuities is one of the areas where feedback is sought in this consultation (Question I).

Face value

Under the proposed design, on each payment date the principal balance outstanding would fall. The Annuity Rate would be set in relation to the original principal balance (i.e. loaned amount) at the first issue date, which would be called the 'face value'. The DMO feels that it is important that all re-openings of annuities would be fully fungible with the parent issue, and invites suggestions from market participants regarding the practicalities in achieving this, including any design modifications that may be desirable.

Compounding convention

The compounding convention would be similar for both types of annuities, and would be based on the convention used for bullet conventional and index-linked gilts[7]. Compounding would occur on quasi-payment dates. Quasi-payment dates are the dates on the payment cycle defined by the maturity date, irrespective of whether cash flows occur on those dates (a quasi-payment date on which a cash flow would not occur is the next quasi-payment date of an annuity settling in its ex-dividend period). A full (quasi-) payment period is defined as the period between two consecutive quasi-payment dates. For example, an annuity settling on its issue date (assuming this is not also a quasi-payment date) would have a quasi-payment period which starts on the quasi-payment date prior to the issue date and ends on the first quasi-payment date following the issue date. Cash flows and quasi-payment dates which are due to occur on non-business days are not adjusted (i.e. they are not 'bumped').

Daycount convention

The daycount convention to be used for both discounting (i.e. price / yield calculations) and accrued interest calculations would be actual / actual.

Ex-dividend period

For the purpose of this annex, it is assumed that annuities would carry the standard ex-dividend period of seven business days. However, views on whether annuities should have a zero ex-dividend period by restricting ownership to holdings in CREST are sought as part of this consultation (Question k).

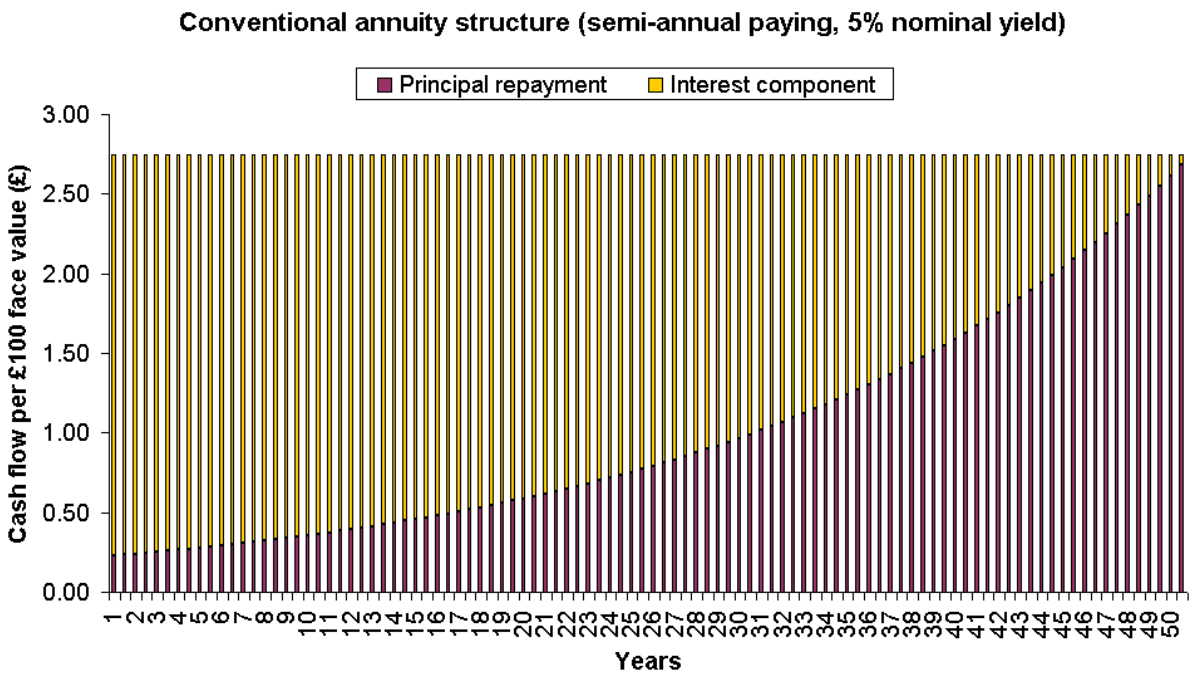
Taxation

The descriptions below show how the cash flows would effectively be composed, although how they would be treated for taxation purposes has yet to be defined. It should not be assumed that the breakdown of the cash flows presented here would necessarily apply for taxation purposes.

[7] See pages 2-3 in 'Formulae for Calculating Gilt Prices from Yields', January 2002, available at http://www.dmo.gov.uk/gilts/public/technical/yldeqns_v2.pdf

Conventional annuities

Rather than paying semi-annual interest followed by a bullet repayment on the maturity date, a conventional annuity would make a regular fixed payment which would include both interest and some repayment of the principal. This (semi-annual) payment per £100 face value would be called the 'Annuity Payment'; the annual rate would be referred to as the 'Annuity Rate'. Hence the principal would be repaid over the life of the bond, and there would be no large final redemption payment on the maturity date. The structure would be similar to a fixed term fixed rate repayment mortgage.



Determination of the Annuity Rate

The Annuity Rate would be calculated by the DMO at the time of first issue and this would remain fixed for the life of the instrument, including following re-openings. This Annuity Rate would be calculated with reference to the quasi-payment date immediately prior to the issue date, rather than from the issue date itself (except in the case where an annuity were to be issued on a quasi-payment date). The following formula would be used for its calculation:

$$A = \frac{100Y}{\left(1 - V^{2T}\right)}$$

where:

A = Annual Annuity Rate, expressed in £ per £100 face value

Y = Semi-annually compounded annual interest rate payable on the loan, expressed as a decimal (i.e. if the interest rate is 5% then Y = 0.05). This is the rate that is applied to the outstanding capital, and is fixed from the first issue of the annuity

V = $\dfrac{1}{\left(1 + \dfrac{Y}{2}\right)}$

T = Number of years from the quasi-payment date prior to the issue date to the maturity date (a multiple of 0.5)

The annual Annuity Rate would be rounded to the nearest convenient fractional unit, say 1/8%, and quoted as a fraction. The naming convention for the annuity would take the form '5½% Treasury Annuity Stock 2051'.

All cash flows would be of equal size in nominal terms, including the first cash flow of the first issue of the annuity. Hence, when first issued, and indeed for subsequent re-openings, an amount of accrued interest would be paid by purchasers to the DMO in addition to the clean price (unless the issue date is on a quasi-payment date). This is in contrast to the convention for issuing new (conventional or index-linked) gilts for the first time, where the first cash flow is non-standard (short or long) and the accrued interest paid is always zero.

The breakdown between the interest and principal repayment at each stage would be determined according to the following.

Further notation

m_t = Principal outstanding per £100 face value after the lapse of t 6-month periods

$m_t = m_{t-1} - p_t$ where $m_0 = 100$ and $m_{2T} = 0$

p_t = Principal repayment per £100 face value at the end of the t-th 6-month period

c_t = Interest payment per £100 face value at the end of the t-th 6-month period

Then:

$$m_t = 100 \times \left(\frac{\left(1+\frac{Y}{2}\right)^{2T} - \left(1+\frac{Y}{2}\right)^{t}}{\left(1+\frac{Y}{2}\right)^{2T} - 1} \right) \quad t \geq 1$$

$$c_t = m_{t-1} \times \frac{Y}{2} \quad t \geq 1$$

Price / yield calculation

It is assumed that annuities would be quoted and traded on a price rather than yield basis.

The price / yield formula would be:

$$P = \frac{Av^{\frac{r}{s}}}{2}\left(A_1 + \frac{2\left(1-v^n\right)}{y}\right) \qquad \text{for all } n \geq 0$$

where:

P = Dirty price per £100 face value

A = Annual Annuity Rate, expressed in £ per £100 face value

A_1 = 0 if the settlement date is in the ex-dividend period; 1 otherwise (including if the settlement date occurs on a quasi-payment date)

y = Semi-annually compounded nominal redemption yield, expressed as a decimal (i.e. if the yield is 5% then $y = 0.05$). Note that this varies during an annuity's life, unlike Y in the formula for calculating the Annuity Rate above

v $= \dfrac{1}{\left(1+\dfrac{y}{2}\right)}$

r = Number of calendar days from the settlement date to the next quasi-payment date ($r = s$ if the settlement date occurs on a quasi-payment date)

s = Number of calendar days in the full quasi-payment period in which the settlement date occurs (i.e. between the prior quasi-payment date and the following quasi-payment date). If the settlement date occurs on a quasi-payment date, s is measured for the quasi-payment period starting on the settlement date

n = Number of full quasi-payment periods between the next quasi-payment date after the settlement date and the maturity date

In the event that the formula were to be used to derive a yield from a price it is not possible (in most cases) to solve for yield in terms of price algebraically, and so some form of numerical technique would need to be used if, given a price, a value for the redemption yield is required[8].

Accrued interest

The accrued interest for conventional annuities would be calculated as follows:

$$AI = \begin{cases} \dfrac{t}{s}\cdot\dfrac{A}{2} & \text{if the settlement date occurs on or before the ex - dividend date} \\ \left(\dfrac{t}{s}-1\right)\cdot\dfrac{A}{2} & \text{if the settlement date occurs after the ex - dividend date} \end{cases}$$

[8] In order to solve some types of equation it is necessary to obtain numerical approximations to the roots using an iterative process. An iterative process starts with an approximation x_0 to a root λ from which another approximation x_1 is obtained, and then another approximation x_2, and so on. For an effective process (for a particular root) the successive values (or iterates) x_1, x_2, x_3,... should become progressively closer to the root λ. The process is continued until an approximation of the required accuracy is obtained.

Where:

AI = Accrued interest per £100 face value

A = Annual Annuity Rate, expressed in £ per £100 face value

t = Number of calendar days from the previous quasi-payment date to the settlement date (t = 0 if the settlement date occurs on a quasi-payment date)

s = Number of calendar days in the full quasi-payment period in which the settlement date occurs

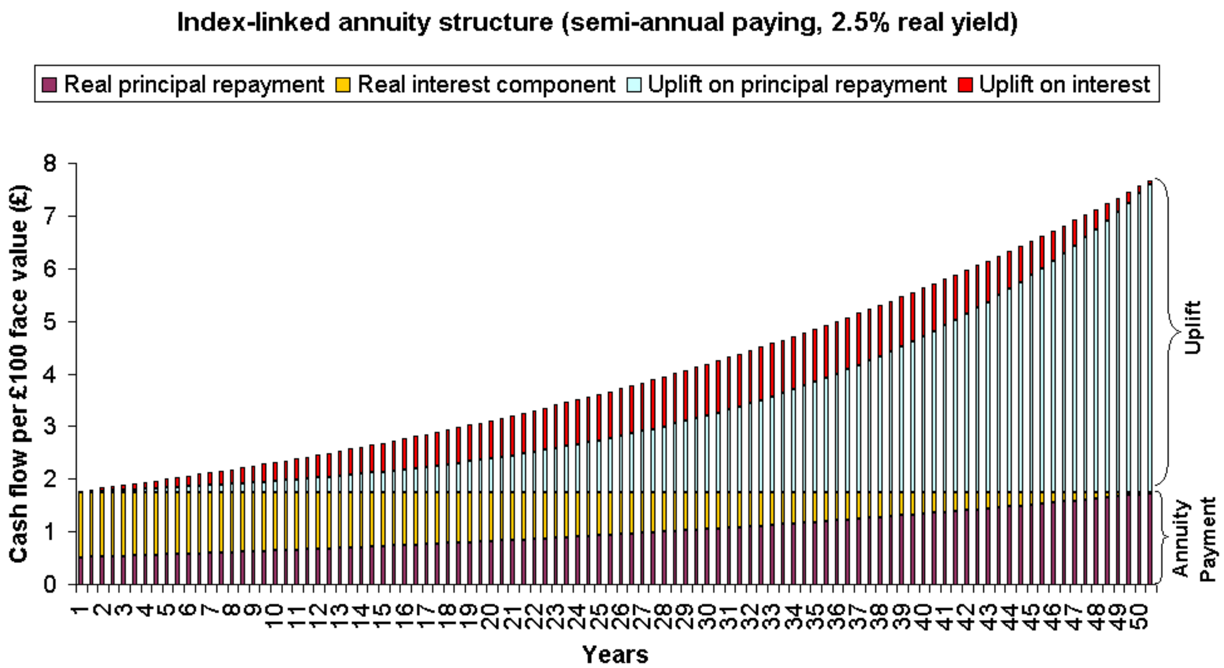
<u>Example cash flows for a conventional annuity</u>

Consider a 50 year conventional annuity of £100 face value first issued on 2 October 2001, paying semi-annually on 2 April and 2 October each year until the final payment on 2 October 2051. Assume that the semi-annually compounded interest rate used for the purpose of calculating the Annuity Rate is 5% per annum. This gives an Annuity Rate of 5.462375…%. This is then rounded to the nearest 1/8%, to 5½%. This rounding gives an effective interest rate which is applied to the outstanding capital of 5.044417…%. Assuming that at the time of issue the market redemption yield used for discounting the cash flows (for calculating the price) is still 5%, then the cash flows for this structure would be as in the following schedule.

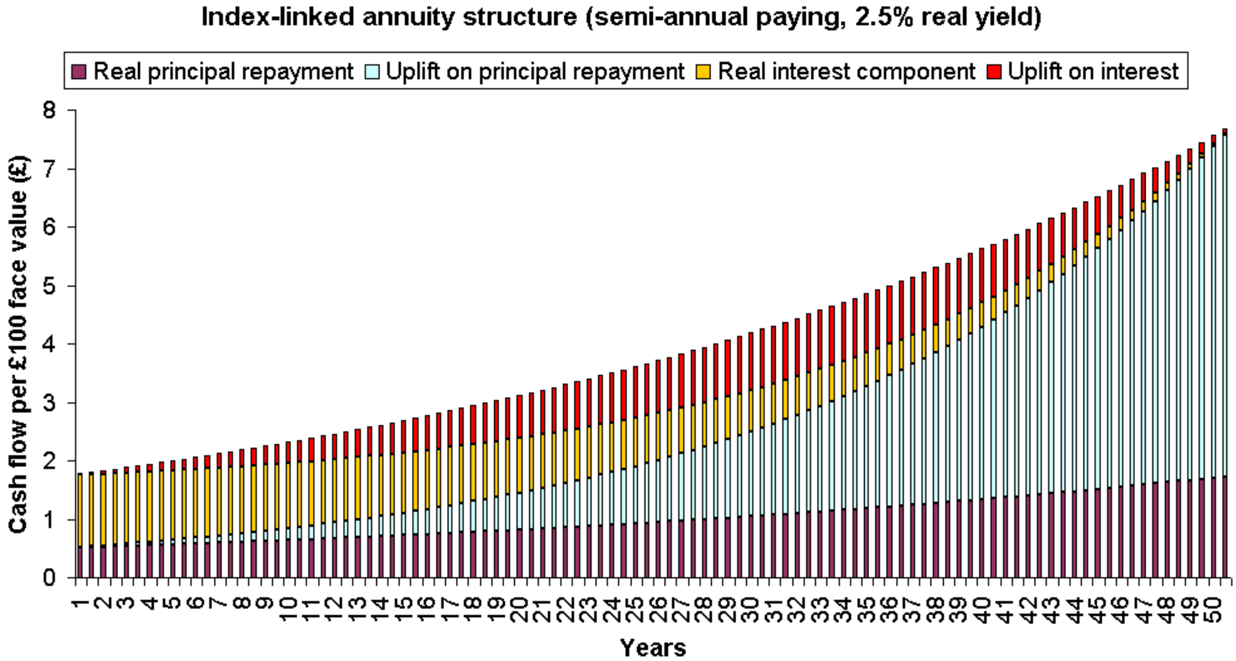
Cash flow date	Time (quasi-payment periods)	Time (years)	Outstanding principal	Principal repayment	Interest component	Total cash flow	Present value of cash flow
02/10/2001			100.000000				
02/04/2002	1.00	0.50	99.772209	0.227791	2.522209	2.750000	2.682927
02/10/2002	2.00	1.00	99.538672	0.233537	2.516463	2.750000	2.617490
02/04/2003	3.00	1.50	99.299245	0.239427	2.510573	2.750000	2.553649
02/10/2003	4.00	2.00	99.053779	0.245466	2.504534	2.750000	2.491364
02/04/2004	5.00	2.50	98.802122	0.251657	2.498343	2.750000	2.430599
02/10/2004	6.00	3.00	98.544118	0.258004	2.491996	2.750000	2.371317
02/04/2005	7.00	3.50	98.279606	0.264512	2.485488	2.750000	2.313480
02/10/2005	8.00	4.00	98.008422	0.271183	2.478817	2.750000	2.257053
02/04/2006	9.00	4.50	97.730399	0.278023	2.471977	2.750000	2.202003
02/10/2006	10.00	5.00	97.445364	0.285035	2.464965	2.750000	2.148296
02/04/2007	11.00	5.50	97.153139	0.292225	2.457776	2.750000	2.095898
02/10/2007	12.00	6.00	96.853544	0.299595	2.450405	2.750000	2.044779
02/04/2008	13.00	6.50	96.546393	0.307152	2.442849	2.750000	1.994906
02/10/2008	14.00	7.00	96.231494	0.314899	2.435102	2.750000	1.946250
02/04/2009	15.00	7.50	95.908653	0.322841	2.427159	2.750000	1.898780
02/10/2009	16.00	8.00	95.577669	0.330984	2.419017	2.750000	1.852469
02/04/2010	17.00	8.50	95.238338	0.339332	2.410668	2.750000	1.807287
02/10/2010	18.00	9.00	94.890447	0.347890	2.402110	2.750000	1.763206
02/04/2011	19.00	9.50	94.533782	0.356665	2.393335	2.750000	1.720201
02/10/2011	20.00	10.00	94.168121	0.365661	2.384339	2.750000	1.678245
02/04/2012	21.00	10.50	93.793238	0.374884	2.375117	2.750000	1.637312
02/10/2012	22.00	11.00	93.408899	0.384339	2.365661	2.750000	1.597378
02/04/2013	23.00	11.50	93.014866	0.394033	2.355968	2.750000	1.558418
02/10/2013	24.00	12.00	92.610895	0.403971	2.346029	2.750000	1.520407
02/04/2014	25.00	12.50	92.196735	0.414160	2.335840	2.750000	1.483324
02/10/2014	26.00	13.00	91.772129	0.424606	2.325394	2.750000	1.447146
02/04/2015	27.00	13.50	91.336814	0.435315	2.314685	2.750000	1.411849
02/10/2015	28.00	14.00	90.890519	0.446295	2.303705	2.750000	1.377414
02/04/2016	29.00	14.50	90.432967	0.457552	2.292449	2.750000	1.343819
02/10/2016	30.00	15.00	89.963875	0.469092	2.280908	2.750000	1.311042
02/04/2017	31.00	15.50	89.482952	0.480923	2.269077	2.750000	1.279066
02/10/2017	32.00	16.00	88.989899	0.493053	2.256947	2.750000	1.247869
02/04/2018	33.00	16.50	88.484409	0.505489	2.244511	2.750000	1.217433
02/10/2018	34.00	17.00	87.966171	0.518239	2.231762	2.750000	1.187740
02/04/2019	35.00	17.50	87.434861	0.531310	2.218691	2.750000	1.158771
02/10/2019	36.00	18.00	86.890151	0.544710	2.205290	2.750000	1.130508
02/04/2020	37.00	18.50	86.331702	0.558449	2.191551	2.750000	1.102934
02/10/2020	38.00	19.00	85.759167	0.572534	2.177466	2.750000	1.076034
02/04/2021	39.00	19.50	85.172192	0.586975	2.163025	2.750000	1.049789
02/10/2021	40.00	20.00	84.570413	0.601780	2.148221	2.750000	1.024184
02/04/2022	41.00	20.50	83.953455	0.616958	2.133042	2.750000	0.999204
02/10/2022	42.00	21.00	83.320936	0.632519	2.117481	2.750000	0.974833
02/04/2023	43.00	21.50	82.672464	0.648472	2.101528	2.750000	0.951057
02/10/2023	44.00	22.00	82.007636	0.664828	2.085172	2.750000	0.927860
02/04/2024	45.00	22.50	81.326039	0.681596	2.068404	2.750000	0.905230
02/10/2024	46.00	23.00	80.627252	0.698788	2.051213	2.750000	0.883151
02/04/2025	47.00	23.50	79.910839	0.716413	2.033588	2.750000	0.861611
02/10/2025	48.00	24.00	79.176357	0.734482	2.015518	2.750000	0.840596
02/04/2026	49.00	24.50	78.423350	0.753007	1.996993	2.750000	0.820093
02/10/2026	50.00	25.00	77.651350	0.772000	1.978001	2.750000	0.800091
02/04/2027	51.00	25.50	76.859879	0.791471	1.958529	2.750000	0.780577
02/10/2027	52.00	26.00	76.048446	0.811434	1.938567	2.750000	0.761538
02/04/2028	53.00	26.50	75.216546	0.831900	1.918101	2.750000	0.742964
02/10/2028	54.00	27.00	74.363665	0.852882	1.897118	2.750000	0.724843
02/04/2029	55.00	27.50	73.489271	0.874393	1.875607	2.750000	0.707164
02/10/2029	56.00	28.00	72.592824	0.896447	1.853553	2.750000	0.689916
02/04/2030	57.00	28.50	71.673766	0.919058	1.830943	2.750000	0.673089
02/10/2030	58.00	29.00	70.731528	0.942238	1.807762	2.750000	0.656672
02/04/2031	59.00	29.50	69.765525	0.966003	1.783997	2.750000	0.640656
02/10/2031	60.00	30.00	68.775157	0.990368	1.759632	2.750000	0.625030
02/04/2032	61.00	30.50	67.759810	1.015347	1.734653	2.750000	0.609785
02/10/2032	62.00	31.00	66.718853	1.040956	1.709044	2.750000	0.594912
02/04/2033	63.00	31.50	65.651642	1.067211	1.682789	2.750000	0.580402
02/10/2033	64.00	32.00	64.557513	1.094129	1.655872	2.750000	0.566246
02/04/2034	65.00	32.50	63.435788	1.121725	1.628275	2.750000	0.552435
02/10/2034	66.00	33.00	62.285771	1.150017	1.599983	2.750000	0.538961
02/04/2035	67.00	33.50	61.106748	1.179023	1.570977	2.750000	0.525816
02/10/2035	68.00	34.00	59.897988	1.208760	1.541240	2.750000	0.512991
02/04/2036	69.00	34.50	58.658740	1.239248	1.510752	2.750000	0.500479
02/10/2036	70.00	35.00	57.388236	1.270504	1.479496	2.750000	0.488272
02/04/2037	71.00	35.50	56.085686	1.302549	1.447451	2.750000	0.476363
02/10/2037	72.00	36.00	54.750284	1.335402	1.414598	2.750000	0.464745
02/04/2038	73.00	36.50	53.381201	1.369084	1.380917	2.750000	0.453409
02/10/2038	74.00	37.00	51.977586	1.403615	1.346385	2.750000	0.442351
02/04/2039	75.00	37.50	50.538569	1.439017	1.310983	2.750000	0.431562
02/10/2039	76.00	38.00	49.063257	1.475312	1.274688	2.750000	0.421036
02/04/2040	77.00	38.50	47.550734	1.512522	1.237478	2.750000	0.410767
02/10/2040	78.00	39.00	46.000063	1.550671	1.199329	2.750000	0.400748
02/04/2041	79.00	39.50	44.410281	1.589783	1.160218	2.750000	0.390974
02/10/2041	80.00	40.00	42.780400	1.629880	1.120120	2.750000	0.381438
02/04/2042	81.00	40.50	41.109411	1.670989	1.079011	2.750000	0.372134
02/10/2042	82.00	41.00	39.396276	1.713135	1.036865	2.750000	0.363058
02/04/2043	83.00	41.50	37.639932	1.756344	0.993656	2.750000	0.354203
02/10/2043	84.00	42.00	35.839290	1.800643	0.949358	2.750000	0.345564
02/04/2044	85.00	42.50	33.993231	1.846058	0.903942	2.750000	0.337135
02/10/2044	86.00	43.00	32.100611	1.892620	0.857380	2.750000	0.328912
02/04/2045	87.00	43.50	30.160256	1.940356	0.809644	2.750000	0.320890
02/10/2045	88.00	44.00	28.170960	1.989296	0.760705	2.750000	0.313064
02/04/2046	89.00	44.50	26.131490	2.039470	0.710530	2.750000	0.305428
02/10/2046	90.00	45.00	24.040581	2.090909	0.659091	2.750000	0.297978
02/04/2047	91.00	45.50	21.896934	2.143647	0.606354	2.750000	0.290711
02/10/2047	92.00	46.00	19.699220	2.197714	0.552286	2.750000	0.283620
02/04/2048	93.00	46.50	17.446076	2.253145	0.496855	2.750000	0.276703
02/10/2048	94.00	47.00	15.136102	2.309974	0.440026	2.750000	0.269954
02/04/2049	95.00	47.50	12.767866	2.368236	0.381764	2.750000	0.263370
02/10/2049	96.00	48.00	10.339898	2.427968	0.322032	2.750000	0.256946
02/04/2050	97.00	48.50	7.850691	2.489206	0.260794	2.750000	0.250679
02/10/2050	98.00	49.00	5.298702	2.551989	0.198011	2.750000	0.244565
02/04/2051	99.00	49.50	2.682346	2.616356	0.133644	2.750000	0.238600
02/10/2051	100.00	50.00	0.000000	2.682346	0.067654	2.750000	0.232780
						TOTAL (= Dirty Price)	**100.688798**

Index-linked annuities

Rather than paying semi-annual interest followed by a bullet repayment on the maturity date, an index-linked annuity would make a regular fixed payment adjusted each time by an Index Ratio, which would include both interest and some repayment of the principal. Hence the principal would be repaid over the life of the bond, and there would be no large final redemption payment on the maturity date. Adjusting each cash flow by an



appropriate Index Ratio means that their real values would be maintained over time, albeit with a small (up to 3 months) indexation lag. The fixed part of the overall (semi-annual) payment per £100 face value would be called the 'Annuity Payment'; the annual rate would be referred to as the 'Annuity Rate'. The inflation component which would be added to the Annuity Payment would be called the 'Uplift'. The combined cash flow, consisting of the Uplift and the Annuity Payment, would be called the 'Uplifted Annuity Payment'.



There are two main ways in which to display the cash flow profile for index-linked annuities, assuming the design is as proposed in this annex. The first figure above shows the cash flows split between the Annuity Payment and uplift components; the second shows the split between total principal repayments and total interest payments (both of these themselves consisting of real and uplift components)[9]. For the purpose of illustration both figures assume a constant positive annual rate of inflation.

Determination of the Annuity Rate

The Annuity Rate would be calculated by the DMO at the time of first issue and this would remain fixed for the life of the instrument, including following re-openings. This Annuity Rate would be calculated with reference to the quasi-payment date immediately prior to the issue date, rather than from the issue date itself (except in the case where an annuity were to be issued on a quasi-payment date). The following formula would be used for its calculation:

[9] The two figures above represent exactly the same instrument.

$$A = \frac{100R}{\left(1 - W^{2T}\right)}$$

where:

A = Annual Annuity Rate, expressed in £ per £100 face value

R = Semi-annually compounded annual real interest rate payable on the loan, expressed as a decimal (i.e. if the real interest rate is 2.5% then R =0.025). This is the rate that is applied to the outstanding capital, and is fixed from the first issue of the annuity

$$W = \frac{1}{\left(1 + \frac{R}{2}\right)}$$

T = Number of years from the quasi-payment date prior to the issue date to the maturity date (a multiple of 0.5)

The annual Annuity Rate would be rounded to the nearest convenient fractional unit, say 1/8%, and quoted as a fraction. The naming convention for the annuity would take the form '3½% Index-linked Treasury Annuity Stock 2051'.

When first issued, and indeed for subsequent re-openings, an amount of accrued interest would be paid by purchasers to the DMO in addition to the clean price (unless the issue date is on a quasi-payment date). This is in contrast to the convention for issuing new (conventional or index-linked) gilts for the first time, where the first cash flow is non-standard (short or long) and the accrued interest paid is always zero.

The breakdown between the interest and principal repayment within the Annuity Payment at each stage would be determined according to the following.

Further notation

m_t = Real principal outstanding per £100 face value after the lapse of *t* 6-month periods

$m_t = m_{t-1} - p_t$ where $m_0 = 100$ and $m_{2T} = 0$

p_t = Real principal repayment per £100 face value at the end of the *t*-th 6-month period

c_t = Real interest payment per £100 face value at the end of the *t*-th 6-month period

Then:

$$m_t = 100 \times \left(\frac{\left(1+\frac{R}{2}\right)^{2T} - \left(1+\frac{R}{2}\right)^{t}}{\left(1+\frac{R}{2}\right)^{2T} - 1} \right) \quad t \geq 1$$

$$c_t = m_{t-1} \times \frac{R}{2} \quad t \geq 1$$

Choice of price index

It is assumed that index-linked annuities would be linked to the Retail Prices Index (RPI).

Indexation methodology

Any index-linked annuity would employ the three-month lag indexation technique first used in the Canadian Real Return Bond (RRB) market, rather than the eight-month lag methodology used for existing index-linked gilts. In addition to using a shorter lag, RRB indexation is applied in a significantly different way from that for existing index-linked gilts.

An Index Ratio would be applied to calculate the uplifted payments and the accrued interest. The Index Ratio for a given date would be defined as the ratio of the reference RPI applicable to a given date (“$\text{Ref RPI}_{\text{Date}}$”) divided by the reference RPI applicable to the original issue date of the gilt (“$\text{Ref RPI}_{\text{First Issue Date}}$”), rounded to the nearest 5th decimal place:

$$\text{Index Ratio}_{\text{Date}} = \left[\frac{\text{Ref RPI}_{\text{Date}}}{\text{Ref RPI}_{\text{First Issue Date}}} \right]$$, rounded to the nearest 5th decimal place.

The reference RPI for the first calendar day of any calendar month would be the RPI for the calendar month falling three months earlier. For example, the reference RPI for 1 June corresponds to the RPI for March, the reference RPI for 1 July corresponds to the RPI for April, etc. The reference RPI for any other day in the month would be calculated by linear interpolation between the reference RPI applicable to the first calendar day of the month in which the day falls and the reference RPI applicable to the first calendar day of the month immediately following. Interpolated values for $\text{Ref RPI}_{\text{Date}}$ would be rounded to the nearest 5th decimal place, as would values for $\text{Index Ratio}_{\text{Date}}$.

The formula used to calculate $\text{Ref RPI}_{\text{Date}}$ would be expressed as follows:

$$\text{Ref RPI}_{\text{Date}} = \text{Ref RPI}_{\text{M}} + \left(\frac{t-1}{D}\right)\left[\text{Ref RPI}_{\text{M+1}} - \text{Ref RPI}_{\text{M}}\right]$$

where:

D = The number of days in the calendar month in which the given date falls

t = The calendar day corresponding to the given date

$\text{Ref RPI}_{\text{M}}$ = Reference RPI for the first day of the calendar month in which the given date falls

$\text{Ref RPI}_{\text{M+1}}$ = Reference RPI for the first day of the calendar month immediately following the given date

For example, the reference RPI for 20 July 2001 would be calculated as follows:

$$\text{Ref RPI}_{\text{20 July 2001}} = \text{Ref RPI}_{\text{1 July 2001}} + \left(\frac{19}{31}\right)\left[\text{Ref RPI}_{\text{1 August 2001}} - \text{Ref RPI}_{\text{1 July 2001}}\right]$$

$$= \text{RPI}_{\text{April 2001}} + \left(\frac{19}{31}\right)\left[\text{RPI}_{\text{May 2001}} - \text{RPI}_{\text{April 2001}}\right]$$

$$= 173.1 + \left(\frac{19}{31}\right)\left[174.2 - 173.1\right] = 173.77419$$, when rounded to the nearest 5th decimal place.

The $\text{Ref RPI}_{\text{First Issue Date}}$ for a given annuity would remain constant over its life. However, different index-linked annuities would have different values for $\text{Ref RPI}_{\text{First Issue Date}}$ (depending on the first issue date).

Uplifted payments

The cash flows or uplifted payments on an index-linked annuity would be calculated by multiplying the Annuity Payment by the Index Ratio applicable to the payment date, rounded to the nearest 6th decimal place per £100 face value.

$$\text{Uplifted Payment}_{\text{Payment Date}} = \text{Annuity Payment} \times \text{Index Ratio}_{\text{Payment Date}}$$

There would be no floor on the level of uplifted payments.

Calculation of the settlement price

Index-linked annuities would most likely trade on the basis of the Real Clean Price per £100 face value (real prices would be quoted to 2 decimal places).

The Inflation-Adjusted Clean Price per £100 face value would be calculated from the real price using the following formula:

$$\text{Inflation-Adjusted Clean Price} = \text{Real Clean Price} \times \text{Index Ratio}_{\text{Set Date}}$$ (this should be left unrounded)

The Inflation-Adjusted Dirty Price per £100 face value would be calculated as:

$$\text{Inflation-Adjusted Dirty Price per £100 face value} = \text{Inflation-Adjusted Clean Price per £100 face value} + \text{Inflation-Adjusted Accrued Interest per £100 face value}$$

(this should be left unrounded)

where: $\text{Inflation-Adjusted Accrued Interest} = \text{Real Accrued Interest} \times \text{Index Ratio}_{\text{Set Date}}$

and the Real Accrued Interest (*RAI*) is defined below. The Inflation-Adjusted Accrued Interest should be left unrounded.

Price / yield calculation

(1) Where the trade occurs before the publication of the RPI that determines the final uplifted payment, the price / yield formula would be given by:

$$P = \frac{Aw^{\frac{r}{s}}}{2}\left(A_1 + \frac{2\left(1 - w^n\right)}{\rho}\right) \qquad \text{for all } n \geq 0$$

where:

P = Real dirty price per £100 face value

A = Annual Annuity Rate, expressed in £ per £100 face value

A_1 = 0 if the settlement date is in the ex-dividend period; 1 otherwise (including if the settlement date occurs on a quasi-payment date)

ρ = Semi-annually compounded real redemption yield, expressed as a decimal (i.e. if the real yield is 2.5% then ρ=0.025). Note that this varies during an annuity's life, unlike R in the formula for calculating the Annuity Rate above

w = $\frac{1}{\left(1+\frac{\rho}{2}\right)}$

r = Number of calendar days from the settlement date to the next quasi-payment date ($r = s$ if the settlement date occurs on a quasi-payment date)

s = Number of calendar days in the full quasi-payment period in which the settlement date occurs (i.e. between the prior quasi-payment date and the following quasi-payment date). If the settlement date occurs on a quasi-payment date, s is measured for the quasi-payment period starting on the settlement date

n = Number of full quasi-payment periods between the next quasi-payment date after the settlement date and the maturity date

In the event that the formula were to be used to derive a yield from a price it is not possible (in most cases) to solve for yield in terms of price algebraically, and so some form of numerical technique would need to be used if, given a price, a value for the redemption yield is required[10].

(2) Where the trade occurs after the publication of the RPI that determines the final uplifted payment, the annuity would effectively become a nominal (rather than real) instrument and the formula for calculating the (real) dirty price from the nominal yield would be given by:

$$P = \left(\frac{1}{\text{Index Ratio}_{\text{Set Date}}} \right) \left[v^{\frac{r}{s}} \left(UP_{LAST} \right) \right]$$

In this case, it is possible to solve algebraically for yield in terms of price:

$$y = 2 \times \left[\left(\frac{P \times \text{Index Ratio}_{\text{Set Date}}}{UP_{LAST}} \right)^{-\frac{s}{r}} - 1 \right]$$

where additional terms are defined as follows:

y = Semi-annually compounded nominal redemption yield (decimal) i.e. if the nominal yield is 5% then y = 0.05

v = $\dfrac{1}{\left(1 + \dfrac{y}{2}\right)}$

UP_{LAST} = Final (fixed) uplifted payment per £100 face value of the gilt, as published

Accrued interest

The real accrued interest for index-linked annuities would be calculated as follows:

$$RAI = \begin{cases} \dfrac{t}{s} \times \dfrac{A}{2} & \text{if the settlement date occurs on or before the ex-dividend date} \\ \left(\dfrac{t}{s} - 1 \right) \times \dfrac{A}{2} & \text{if the settlement date occurs after the ex-dividend date} \end{cases}$$

[10] See footnote 8.

where:

RAI = Real accrued interest per £100 face value

A = Annual Annuity Rate, expressed in £ per £100 face value

t = Number of calendar days from the previous quasi-payment date to the settlement date ($t = 0$ if the settlement date occurs on a quasi-payment date)

s = Number of calendar days in the full quasi-payment period in which the settlement date occurs

The Inflation-Adjusted Accrued Interest would be calculated as:

$$\text{Inflation-Adjusted Accrued Interest} = \text{Real Accrued Interest} \times \text{Index Ratio}_{\text{Set Date}}$$

<u>Example cash flows for an index-linked annuity</u>

Consider a 50 year index-linked annuity of £100 face value first issued on 2 October 2001, paying semi-annually on 2 April and 2 October each year until the final payment on 2 October 2051. Assume that the semi-annually compounded real interest rate used for the purpose of calculating the Annuity Rate is 2.5% per annum. This gives an Annuity Rate of 3.514855…%. This is then rounded to the nearest 1/8%, to 3½%. This rounding gives an effective real interest rate which is applied to the outstanding capital of 2.478795…%. Assuming that at the time of issue the market real redemption yield used for discounting the cash flows (for calculating the price) is still 2.5%, then the cash flows for this structure would be as in the following schedule.

(Note that for calculating the Real Dirty Price, the *Annuity Payment* (not Uplifted Payment) at each payment date is discounted, using the real yield 2.5%).

Cash flow date	Time (quasi-payment periods)	Time (years)	Ref RPI (3% inflation assumption)	Index Ratio	Outstanding principal	Real principal repayment	Real interest component	Annuity Payment	Uplift on principal repayment	Uplift on interest	Total uplift	Uplifted payment	Present value of Annuity Payment
02/10/2001			173.32258	1.00000	100.000000								
02/04/2002	1.00	0.50	175.90321	1.01489	99.489398	0.510602	1.239398	1.750000	0.007603	0.018455	0.026057	1.776057	1.728395
02/10/2002	2.00	1.00	178.52226	1.03000	98.972467	0.516931	1.233069	1.750000	0.015508	0.036992	0.052500	1.802500	1.707057
02/04/2003	3.00	1.50	181.18030	1.04534	98.449129	0.523338	1.226662	1.750000	0.023728	0.055617	0.079345	1.829345	1.685982
02/10/2003	4.00	2.00	183.87793	1.06090	97.919306	0.529824	1.220176	1.750000	0.032266	0.074309	0.106575	1.856575	1.665167
02/04/2004	5.00	2.50	186.61571	1.07670	97.382915	0.536390	1.213609	1.750000	0.041141	0.093084	0.134225	1.884225	1.644610
02/10/2004	6.00	3.00	189.39426	1.09273	96.839877	0.543038	1.206961	1.750000	0.050356	0.111922	0.162277	1.912277	1.624306
02/04/2005	7.00	3.50	192.21418	1.10900	96.290108	0.549769	1.200231	1.750000	0.059925	0.130825	0.190750	1.940750	1.604253
02/10/2005	8.00	4.00	195.07609	1.12551	95.733525	0.556583	1.193417	1.750000	0.069857	0.149786	0.219642	1.969642	1.584447
02/04/2006	9.00	4.50	197.98061	1.14227	95.170044	0.563481	1.186519	1.750000	0.080166	0.168806	0.248972	1.998972	1.564886
02/10/2006	10.00	5.00	200.92837	1.15927	94.599580	0.570465	1.179535	1.750000	0.090858	0.187865	0.278722	2.028722	1.545567
02/04/2007	11.00	5.50	203.92003	1.17653	94.022044	0.577535	1.172465	1.750000	0.101952	0.206975	0.308927	2.058927	1.526485
02/10/2007	12.00	6.00	206.95622	1.19405	93.437352	0.584693	1.165307	1.750000	0.113460	0.226128	0.339587	2.089587	1.507640
02/04/2008	13.00	6.50	210.03763	1.21183	92.845412	0.591940	1.158060	1.750000	0.125391	0.245312	0.370702	2.120702	1.489027
02/10/2008	14.00	7.00	213.16491	1.22987	92.246136	0.599276	1.150724	1.750000	0.137756	0.264517	0.402272	2.152272	1.470644
02/04/2009	15.00	7.50	216.33876	1.24819	91.639432	0.606704	1.143296	1.750000	0.150578	0.283755	0.434332	2.184332	1.452488
02/10/2009	16.00	8.00	219.55986	1.26677	91.025209	0.614223	1.135777	1.750000	0.163856	0.302991	0.466847	2.216847	1.434556
02/04/2010	17.00	8.50	222.82892	1.28563	90.403373	0.621836	1.128164	1.750000	0.177615	0.322238	0.499852	2.249852	1.416845
02/10/2010	18.00	9.00	226.14665	1.30477	89.773831	0.629543	1.120457	1.750000	0.191866	0.341482	0.533347	2.283347	1.399354
02/04/2011	19.00	9.50	229.51379	1.32420	89.136485	0.637345	1.112655	1.750000	0.206627	0.360723	0.567350	2.317350	1.382078
02/10/2011	20.00	10.00	232.93105	1.34392	88.491241	0.645244	1.104755	1.750000	0.221912	0.379947	0.601860	2.351860	1.365015
02/04/2012	21.00	10.50	236.39920	1.36393	87.837999	0.653242	1.096758	1.750000	0.237734	0.399143	0.636877	2.386877	1.348163
02/10/2012	22.00	11.00	239.91899	1.38423	87.176661	0.661338	1.088662	1.750000	0.254106	0.418297	0.672402	2.422402	1.331519
02/04/2013	23.00	11.50	243.49118	1.40484	86.507127	0.669534	1.080465	1.750000	0.271054	0.437416	0.708470	2.458470	1.315080
02/10/2013	24.00	12.00	247.11656	1.42576	85.829294	0.677833	1.072167	1.750000	0.288594	0.456486	0.745080	2.495080	1.298845
02/04/2014	25.00	12.50	250.79591	1.44699	85.143061	0.686234	1.063766	1.750000	0.306740	0.475493	0.782232	2.532232	1.282810
02/10/2014	26.00	13.00	254.53005	1.46853	84.448322	0.694739	1.055261	1.750000	0.325506	0.494421	0.819927	2.569927	1.266973
02/04/2015	27.00	13.50	258.31979	1.49040	83.744972	0.703349	1.046650	1.750000	0.344923	0.513277	0.858200	2.608200	1.251331
02/10/2015	28.00	14.00	262.16595	1.51259	83.032905	0.712067	1.037933	1.750000	0.364998	0.532034	0.897032	2.647032	1.235882
02/04/2016	29.00	14.50	266.06938	1.53511	82.312013	0.720892	1.029108	1.750000	0.385757	0.550686	0.936442	2.686442	1.220625
02/10/2016	30.00	15.00	270.03093	1.55797	81.582186	0.729827	1.020173	1.750000	0.407221	0.569226	0.976447	2.726447	1.205555
02/04/2017	31.00	15.50	274.05147	1.58116	80.843314	0.738872	1.011128	1.750000	0.429403	0.587627	1.017030	2.767030	1.190672
02/10/2017	32.00	16.00	278.13186	1.60471	80.095284	0.748030	1.001970	1.750000	0.452341	0.605901	1.058242	2.808242	1.175972
02/04/2018	33.00	16.50	282.27301	1.62860	79.337983	0.757301	0.992699	1.750000	0.476039	0.624011	1.100050	2.850050	1.161454
02/10/2018	34.00	17.00	286.47582	1.65285	78.571297	0.766687	0.983313	1.750000	0.500532	0.641956	1.142487	2.892487	1.147115
02/04/2019	35.00	17.50	290.74120	1.67746	77.795107	0.776189	0.973811	1.750000	0.525837	0.659718	1.185555	2.935555	1.132953
02/10/2019	36.00	18.00	295.07009	1.70243	77.009298	0.785809	0.964191	1.750000	0.551976	0.677276	1.229252	2.979252	1.118966
02/04/2020	37.00	18.50	299.46344	1.72778	76.213750	0.795549	0.954451	1.750000	0.578984	0.694631	1.273615	3.023615	1.105152
02/10/2020	38.00	19.00	303.92219	1.75351	75.408341	0.805409	0.944591	1.750000	0.606883	0.711759	1.318642	3.068642	1.091508
02/04/2021	39.00	19.50	308.44734	1.77961	74.592950	0.815391	0.934609	1.750000	0.635687	0.728631	1.364317	3.114317	1.078032
02/10/2021	40.00	20.00	313.03986	1.80611	73.767454	0.825497	0.924503	1.750000	0.665441	0.745251	1.410692	3.160692	1.064723
02/04/2022	41.00	20.50	317.70076	1.83300	72.931726	0.835728	0.914272	1.750000	0.696161	0.761589	1.457750	3.207750	1.051579
02/10/2022	42.00	21.00	322.43105	1.86029	72.085640	0.846086	0.903914	1.750000	0.727879	0.777628	1.505507	3.255507	1.038596
02/04/2023	43.00	21.50	327.23178	1.88799	71.229068	0.856572	0.893428	1.750000	0.760628	0.793355	1.553982	3.303982	1.025774
02/10/2023	44.00	22.00	332.10399	1.91610	70.361879	0.867189	0.882811	1.750000	0.794431	0.808743	1.603175	3.353175	1.013110
02/04/2024	45.00	22.50	337.04873	1.94463	69.483942	0.877937	0.872063	1.750000	0.829325	0.823777	1.653102	3.403102	1.000603
02/10/2024	46.00	23.00	342.06711	1.97359	68.595125	0.888818	0.861182	1.750000	0.865344	0.838438	1.703782	3.453782	0.988249
02/04/2025	47.00	23.50	347.16020	2.00297	67.695291	0.899834	0.850166	1.750000	0.902506	0.852691	1.755197	3.505197	0.976049
02/10/2025	48.00	24.00	352.32912	2.03279	66.784305	0.910986	0.839014	1.750000	0.940857	0.866525	1.807382	3.557382	0.963999
02/04/2026	49.00	24.50	357.57500	2.06306	65.862028	0.922277	0.827723	1.750000	0.980436	0.879919	1.860355	3.610355	0.952098

02/10/2026	50.00	25.00	362.89899	2.09378	64.928321	0.933708	0.816292	1.750000	1.021271	0.892844	1.914115	3.664115	0.940343
02/04/2027	51.00	25.50	368.30225	2.12495	63.983041	0.945280	0.804720	1.750000	1.063393	0.905270	1.968662	3.718662	0.928734
02/10/2027	52.00	26.00	373.78596	2.15659	63.026045	0.956996	0.793004	1.750000	1.106852	0.917181	2.024032	3.774032	0.917268
02/04/2028	53.00	26.50	379.35132	2.18870	62.057188	0.968857	0.781143	1.750000	1.151680	0.928545	2.080225	3.830225	0.905944
02/10/2028	54.00	27.00	384.99954	2.22129	61.076324	0.980865	0.769135	1.750000	1.197920	0.939337	2.137257	3.887257	0.894759
02/04/2029	55.00	27.50	390.73186	2.25436	60.083302	0.993021	0.756978	1.750000	1.245606	0.949524	2.195130	3.945130	0.883713
02/10/2029	56.00	28.00	396.54953	2.28793	59.077973	1.005329	0.744671	1.750000	1.294793	0.959084	2.253877	4.003877	0.872803
02/04/2030	57.00	28.50	402.45382	2.32199	58.060185	1.017789	0.732211	1.750000	1.345507	0.967976	2.313482	4.063482	0.862028
02/10/2030	58.00	29.00	408.44601	2.35657	57.029781	1.030403	0.719597	1.750000	1.397814	0.976183	2.373997	4.123997	0.851385
02/04/2031	59.00	29.50	414.52743	2.39165	55.986607	1.043174	0.706826	1.750000	1.451733	0.983654	2.435387	4.185387	0.840874
02/10/2031	60.00	30.00	420.69939	2.42726	54.930504	1.056103	0.693897	1.750000	1.507334	0.990371	2.497705	4.247705	0.830493
02/04/2032	61.00	30.50	426.96325	2.46340	53.861311	1.069193	0.680807	1.750000	1.564656	0.996293	2.560950	4.310950	0.820240
02/10/2032	62.00	31.00	433.32038	2.50008	52.778867	1.082444	0.667556	1.750000	1.623753	1.001387	2.625140	4.375140	0.810114
02/04/2033	63.00	31.50	439.77215	2.53730	51.683007	1.095860	0.654140	1.750000	1.684665	1.005609	2.690275	4.440275	0.800112
02/10/2033	64.00	32.00	446.31999	2.57508	50.573565	1.109442	0.640558	1.750000	1.747460	1.008930	2.756390	4.506390	0.790235
02/04/2034	65.00	32.50	452.96532	2.61342	49.450373	1.123192	0.626808	1.750000	1.812181	1.011304	2.823485	4.573485	0.780479
02/10/2034	66.00	33.00	459.70959	2.65234	48.313259	1.137113	0.612887	1.750000	1.878898	1.012697	2.891595	4.641595	0.770843
02/04/2035	67.00	33.50	466.55427	2.69183	47.162053	1.151207	0.598793	1.750000	1.947646	1.013057	2.960702	4.710702	0.761326
02/10/2035	68.00	34.00	473.50087	2.73191	45.996578	1.165475	0.584525	1.750000	2.018497	1.012345	3.030842	4.780842	0.751927
02/04/2036	69.00	34.50	480.55090	2.77258	44.816659	1.179919	0.570080	1.750000	2.091502	1.010513	3.102015	4.852015	0.742644
02/10/2036	70.00	35.00	487.70590	2.81386	43.622116	1.194543	0.555457	1.750000	2.166734	1.007520	3.174255	4.924255	0.733476
02/04/2037	71.00	35.50	494.96743	2.85576	42.412767	1.209348	0.540651	1.750000	2.244261	1.003319	3.247580	4.997580	0.724421
02/10/2037	72.00	36.00	502.33708	2.89828	41.188430	1.224337	0.525663	1.750000	2.324135	0.997855	3.321990	5.071990	0.715477
02/04/2038	73.00	36.50	509.81645	2.94143	39.948918	1.239512	0.510488	1.750000	2.406425	0.991077	3.397502	5.147502	0.706644
02/10/2038	74.00	37.00	517.40719	2.98523	38.694044	1.254874	0.495126	1.750000	2.491213	0.982939	3.474152	5.224152	0.697920
02/04/2039	75.00	37.50	525.11095	3.02967	37.423618	1.270427	0.479573	1.750000	2.578547	0.973375	3.551922	5.301922	0.689304
02/10/2039	76.00	38.00	532.92941	3.07478	36.137445	1.286173	0.463827	1.750000	2.668525	0.962340	3.630865	5.380865	0.680794
02/04/2040	77.00	38.50	540.86427	3.12056	34.835332	1.302113	0.447887	1.750000	2.761209	0.949770	3.710980	5.460980	0.672389
02/10/2040	78.00	39.00	548.91729	3.16703	33.517080	1.318252	0.431748	1.750000	2.856691	0.935611	3.792302	5.542302	0.664088
02/04/2041	79.00	39.50	557.09020	3.21418	32.182490	1.334590	0.415410	1.750000	2.955023	0.919792	3.874815	5.624815	0.655889
02/10/2041	80.00	40.00	565.38481	3.26204	30.831359	1.351131	0.398869	1.750000	3.056312	0.902258	3.958570	5.708570	0.647792
02/04/2042	81.00	40.50	573.80291	3.31061	29.463482	1.367877	0.382123	1.750000	3.160630	0.882937	4.043567	5.793567	0.639794
02/10/2042	82.00	41.00	582.34635	3.35990	28.078652	1.384830	0.365170	1.750000	3.268061	0.861764	4.129825	5.879825	0.631896
02/04/2043	83.00	41.50	591.01700	3.40993	26.676658	1.401994	0.348006	1.750000	3.378707	0.838670	4.217377	5.967377	0.624095
02/10/2043	84.00	42.00	599.81674	3.46070	25.257288	1.419370	0.330630	1.750000	3.492644	0.813581	4.306225	6.056225	0.616390
02/04/2044	85.00	42.50	608.74751	3.51222	23.820327	1.436962	0.313038	1.750000	3.609964	0.786421	4.396385	6.146385	0.608780
02/10/2044	86.00	43.00	617.81124	3.56452	22.365555	1.454771	0.295229	1.750000	3.730790	0.757120	4.487910	6.237910	0.601264
02/04/2045	87.00	43.50	627.00993	3.61759	20.892753	1.472802	0.277198	1.750000	3.855191	0.725591	4.580782	6.330782	0.593841
02/10/2045	88.00	44.00	636.34558	3.67145	19.401698	1.491056	0.258944	1.750000	3.983281	0.691757	4.675037	6.425037	0.586510
02/04/2046	89.00	44.50	645.82023	3.72612	17.892162	1.509536	0.240464	1.750000	4.115176	0.655534	4.770710	6.520710	0.579269
02/10/2046	90.00	45.00	655.43595	3.78160	16.363917	1.528245	0.221755	1.750000	4.250966	0.616834	4.867800	6.617800	0.572117
02/04/2047	91.00	45.50	665.19484	3.83790	14.816731	1.547186	0.202814	1.750000	4.390759	0.575566	4.966325	6.716325	0.565054
02/10/2047	92.00	46.00	675.09903	3.89504	13.250369	1.566362	0.183638	1.750000	4.534680	0.531640	5.066320	6.816320	0.558078
02/04/2048	93.00	46.50	685.15068	3.95304	11.664594	1.585775	0.164225	1.750000	4.682857	0.484962	5.167820	6.917820	0.551188
02/10/2048	94.00	47.00	695.35200	4.01190	10.059165	1.605429	0.144571	1.750000	4.835392	0.435432	5.270825	7.020825	0.544384
02/04/2049	95.00	47.50	705.70520	4.07163	8.433838	1.625327	0.124673	1.750000	4.992403	0.382949	5.375352	7.125352	0.537663
02/10/2049	96.00	48.00	716.21256	4.13225	6.788367	1.645471	0.104529	1.750000	5.154027	0.327410	5.481437	7.231437	0.531025
02/04/2050	97.00	48.50	726.87636	4.19378	5.122502	1.665865	0.084135	1.750000	5.320407	0.268708	5.589115	7.339115	0.524469
02/10/2050	98.00	49.00	737.69893	4.25622	3.435990	1.686512	0.063488	1.750000	5.491653	0.206731	5.698385	7.448385	0.517994
02/04/2051	99.00	49.50	748.68265	4.31959	1.728576	1.707414	0.042586	1.750000	5.667916	0.141367	5.809282	7.559282	0.511599
02/10/2051	100.00	50.00	759.82990	4.38391	0.000000	1.728576	0.021424	1.750000	5.849346	0.072497	5.921842	7.671842	0.505283
												TOTAL (= Real Dirty Price)	**99.577339**

United Kingdom
Debt
Management
Office

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