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DEBT MANAGEMENT OFFICE

Formulae for Calculating Gilt Prices from Yields

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INTRODUCTION

This paper sets out the United Kingdom Debt Management Office's (DMO) formulae for calculating gilt prices from gross redemption yields, thus allowing a formal settlement convention to be applied to trades conducted on a yield basis. The formulae in this paper have been effective since 1 November 1998. This is the third edition of the paper first published in June 1998. On 2 December 2004, the DMO announced that all new index-linked gilts issued from the 2005-06 financial year would follow the three-month indexation lag methodology which has become the global standard. In this updated version of the paper, the formulae for this design of new index-linked gilts have been added, and other minor changes have been made although the formulae themselves are unchanged.

In the event that the formulae are to be used to derive yields from prices it is not possible (in most cases) to solve for yield in terms of price algebraically, and so some form of numerical technique[1] must be used if, given a price, a value for the redemption yield is required.

The first section of the paper sets out the DMO's price / yield formulae; these are split into the different classes of gilt (new formulae for new instruments will be added to the paper as and when required). For the purpose of this paper, 'cash flows' refer to cash flows receivable by the buyer of the gilt. Also, 'nearest rounding' to, say, six decimal places means round the sixth decimal place up by one if the seventh decimal place is five or above, and then truncate at the sixth decimal place.

Coupon payments on all current gilts outstanding are made semi-annually, with the exception of three undated gilts which pay quarterly: 2½% Annuities, 2¾% Annuities and 2½% Consolidated Stock.

Compounding will occur on quasi-coupon dates. For dated gilts, quasi-coupon dates are the dates on the semi-annual cycle defined by the (final) maturity date, irrespective of whether cash flows occur on those dates (examples of quasi-coupon dates on which cash flows would not occur include the first quasi-coupon date of a new issue having a long first dividend period; the next quasi-coupon date of a gilt settling in its ex-dividend period; and most quasi-coupon dates of a strip). The quasi-coupon dates for undated gilts are defined by their regular coupon cycle. A full quasi-coupon period is defined as the period between two consecutive quasi-coupon dates. For example, a gilt settling on its issue date (assuming this is not also a quasi-coupon date) will have a quasi-coupon period which starts on the quasi-coupon date prior to the issue date and ends on the first quasi-coupon date following the issue date. If the issue date falls on a quasi-coupon date, then the quasi-coupon period starts on the issue date. Cash flows and quasi-coupon dates which are due to occur on non-business days are not adjusted (i.e. are not 'bumped').

This means that cash flows which occur on dates which are not quasi-coupon dates (such as some early redemption payments on double-dated or undated gilts) may have an additional fractional period associated with their discounting process to allow for discounting back (i.e. towards the settlement date) by a fractional period to the quasi-coupon date immediately prior to their occurrence, before being discounted back to the settlement date.

All settlement values derived from these formulae (yield to price) should be rounded to the nearest penny on the trade, with no intermediate rounding. In addition, the price / yield formulae discount all cash flows on the quasi-coupon cycle using the 'actual / actual' daycount convention: this is consistent with the agreed market consensus for discounting the cash flow from a strip.

Following market consultation, it was agreed that the RPI inflation assumption that should be used in the formulae for index-linked gilts with an 8-month indexation lag is 3% per annum. This will be reviewed by the DMO as and when a majority of market participants judge that a review is necessary.

The second section in this paper sets out the formulae for calculating dividend payments on gilts; and the third section provides those for the calculation of accrued interest. Annex A describes the procedure for estimating the nominal values of unknown future cash flows on index-linked gilts with an 8-month indexation lag. Annex B sets out the method of indexation for index-linked gilts first issued from 2005-06.

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SECTION ONE: PRICE / YIELD FORMULAE

Conventional Gilts; Double-dated and Undated Gilts with Assumed (or Actual) Redemption on a Quasi-Coupon Date[2]

The formula for calculating the price from the yield is given by:

$$P = v^{\frac{r}{s}}\left(d_1 + d_2 v + \frac{cv^2}{f(1-v)}(1-v^{n-1}) + 100v^n\right) \qquad \text{for } n \geq 1$$

Where:

P = Dirty price per £100 nominal of the gilt[3].

d_1 = Cash flow due on next quasi-coupon date, per £100 nominal of the gilt (may be zero if the gilt has a long first dividend period or if the gilt settles in its ex-dividend period; or may be greater or less than $\frac{c}{f}$ during long or short first dividend periods respectively).

d_2 = Cash flow due on next but one quasi-coupon date, per £100 nominal of the gilt (may be greater than $\frac{c}{f}$ during long first dividend periods).

c = Coupon per £100 nominal of the gilt.

y = Nominal redemption yield (decimal), i.e. if the yield is 5% then $y = 0.05$.

f = Number of coupons payable on the gilt per year (f will be equal to 2 or 4).

v = $\dfrac{1}{1+\frac{y}{f}}$

r = Number of calendar days from the settlement date to the next quasi-coupon date ($r = s$ if the settlement date falls on a quasi-coupon date).

s = Number of calendar days in the full quasi-coupon period in which the settlement date occurs (i.e. between the prior quasi-coupon date and the following quasi-coupon date).

n = Number of full quasi-coupon periods from the next quasi-coupon date after the settlement date to the redemption date.

For $n = 0$, the equation reduces to

$$P = v^{\frac{r}{s}}(d_1 + 100)$$

In this special case, we can solve algebraically for yield in terms of price:

$$y = f \cdot \left[\left(\frac{d_1 + 100}{P} \right)^{\frac{s}{r}} - 1 \right]$$

Index-linked Gilts (8-Month Indexation Lag)[4]

(1) Not all the nominal values of future cash flows are fixed

Case 1: *Two or more cash flows remaining*

The formula for calculating the price from the yield is given by:

$$P = \left[d_1 + d_2(uw) + \frac{acw^2}{2(1-w)}(1-w^{n-1}) \right] (uw)^{\frac{r}{s}} + 100au^{\frac{r}{s}}w^{\frac{r}{s}+n} \qquad \text{for } n \geq 1$$

Where: P = Dirty price per £100 nominal of the gilt[3].

d_1 = Cash flow due on next quasi-coupon date, per £100 nominal of the gilt (may be zero if the gilt has a long first dividend period or if the gilt settles in its ex-dividend period; or may be greater or less than $\frac{c}{2}$ times the RPI Ratio during long or short first dividend periods respectively).

d_2 = Cash flow due on next but one quasi-coupon date, per £100 nominal of the gilt (may be greater than $\frac{c}{2}$ times the RPI Ratio during long first dividend periods)[5].

c = Coupon per £100 nominal.

r = Number of calendar days from the settlement date to the next quasi-coupon date ($r = s$ if the settlement date falls on a quasi-coupon date).

s = Number of calendar days in the full quasi-coupon period in which the settlement date occurs (i.e. between the prior quasi-coupon date and the following quasi-coupon date).

ρ = Semi-annually compounded real redemption yield (decimal), i.e. if the real yield is 2.5% then ρ = 0.025.

$w = \frac{1}{1+\frac{\rho}{2}}$

π = The assumed annual inflation rate (decimal) = 0.03.

u = $\left(\frac{1}{1+\pi}\right)^{\frac{1}{2}} = \left(\frac{1}{1.03}\right)^{\frac{1}{2}}$

n = Number of full quasi-coupon periods from the next quasi-coupon date after the settlement date to the redemption date.

RPIB = The Base RPI for the gilt, i.e. the RPI scheduled to be published seven months prior to the month of first issue of the gilt and relating to the month eight months prior to the month of first issue of the gilt (for example, if the gilt is first issued in November then its Base RPI is the RPI for March of that year).

RPIL = The latest published RPI at the time of valuation.

k = Number of months between the month of the RPI that defines the dividend due (or would ordinarily be due, in the case of a long first dividend or a gilt settling in its ex-dividend period) on the next quasi-coupon date and the month of the latest published RPI at the time of valuation. For example, if the RPI for January is the RPI that defines the dividend due (or would ordinarily be due, in the case of a long first dividend or a gilt settling in its ex-dividend period) on the next quasi-coupon date and the latest published RPI at the time of valuation is the RPI for April, then k = 3.

a = $\frac{RPIL}{RPIB} \cdot u^{\frac{2k}{12}}$

Case 2: One cash flow remaining (i.e. the final dividend and redemption payment)

If the RPI determining the redemption value is published after the gilt goes ex-dividend for the penultimate time, the price / yield formula is defined as:

$$P = \left(100 + \frac{c}{2}\right) \cdot \frac{a}{u} \cdot (uw)^{\frac{r}{s}+\alpha}$$

Where: P = Dirty price per £100 nominal of the gilt[3].

c = Coupon per £100 nominal.

ρ = Real redemption yield (decimal), i.e. if the yield is 2.5% then ρ = 0.025.

w = $\frac{1}{1+\frac{\rho}{2}}$

π = The assumed annual inflation rate (decimal) = 0.03.

u = $\left(\frac{1}{1+\pi}\right)^{\frac{1}{2}} = \left(\frac{1}{1.03}\right)^{\frac{1}{2}}$

r = Number of calendar days from the settlement date to the next quasi-coupon date ($r = s$ if the settlement date falls on a quasi-coupon date).

s = Number of calendar days in the full quasi-coupon period in which the settlement date occurs (i.e. between the prior quasi-coupon date and the following quasi-coupon date).

$$\alpha = \begin{cases} 1 & \text{if the gilt is settling in its penultimate ex-dividend period} \\ 0 & \text{if the gilt is settling on or after its penultimate quasi-coupon date} \end{cases}$$

$RPIB$ = The Base RPI for the gilt, i.e. the RPI scheduled to be published seven months prior to the month of first issue of the gilt and relating to the month eight months prior to the month of first issue of the gilt (for example, if the gilt is first issued in November then its Base RPI is the RPI for March of that year).

$RPIL$ = The latest published RPI at the time of valuation.

k = Number of months between the month of the RPI that defines the dividend due (or would ordinarily be due, in the case of a long first dividend or a gilt settling in its ex-dividend period) on the next quasi-coupon date and the month of the latest published RPI at the time of valuation. For example, if the RPI for January is the RPI that defines the dividend due (or would ordinarily be due, in the case of a long first dividend or a gilt settling in its ex-dividend period) on the next quasi-coupon date and the latest published RPI at the time of valuation is the RPI for April, then k = 3.

a = $\frac{RPIL}{RPIB} \cdot u^{\frac{2k}{12}}$

In this special case, we can solve algebraically for yield in terms of price:

$$\rho = 2 \cdot \left(u \cdot \left(\frac{\left(100 + \frac{c}{2}\right) \cdot a}{uP} \right)^{\frac{s}{r+\alpha s}} - 1 \right)$$

(2) Nominal values of all future cash flows are fixed

Case 1: Index-linked gilts that have passed both their penultimate ex-dividend date and the point at which the RPI determining the final redemption payment is published provide a known cash flow on just one remaining date. The price / yield formula in this case is:

$$P = v^{\frac{r}{s}+\alpha}\left(d_{LAST} + R\right)$$

Where: P = Dirty price per £100 nominal of the gilt[3].

d_{LAST} = Final dividend payment per £100 nominal of the gilt, as published.

R = Final redemption payment per £100 nominal of the gilt, as published.

y = Semi-annually compounded nominal redemption yield (decimal), i.e. if the yield is 5% then $y = 0.05$.

$v = \frac{1}{1+\frac{y}{2}}$

r = Number of calendar days from the settlement date to the next quasi-coupon date ($r = s$ if the settlement date falls on a quasi-coupon date).

s = Number of calendar days in the full quasi-coupon period in which the settlement date occurs (i.e. between the prior quasi-coupon date and the following quasi-coupon date).

$$\alpha = \begin{cases} 1 & \text{if the gilt is settling in its penultimate ex-dividend period} \\ 0 & \text{if the gilt is settling on or after its penultimate quasi-coupon date} \end{cases}$$

In this special case, we can solve algebraically for yield in terms of price:

$$y = 2 \cdot \left[\left(\frac{d_{LAST} + R}{P} \right)^{\frac{s}{r + \alpha s}} - 1 \right]$$

Case 2: When valuing index-linked gilts between the publication of the RPI determining the redemption payment and the penultimate ex-dividend date (assuming that the RPI determining the redemption value is published before the gilt goes ex-dividend for the penultimate time), the price / yield formula is defined as:

$$P = \left(d_{PEN} + \left(d_{LAST} + R \right) \cdot v \right) \cdot v^{\frac{r}{s}}$$

Where:

- P = Dirty price per £100 nominal of the gilt[3].
- d_{PEN} = Penultimate dividend payment per £100 nominal of the gilt, as published.
- d_{LAST} = Final dividend payment per £100 nominal of the gilt, as published.
- R = Redemption payment per £100 nominal of the gilt, as published.
- y = Semi-annually compounded nominal redemption yield (decimal), i.e. if the yield is 5% then y = 0.05.
- v = $\frac{1}{1 + \frac{y}{2}}$
- r = Number of calendar days from the settlement date to the next quasi-coupon date.
- s = Number of calendar days in the full quasi-coupon period in which the settlement date occurs (i.e. between the prior quasi-coupon date and the following quasi-coupon date).

Index-linked Gilts (3-Month Indexation Lag)[4]

(1) For trades settling before the penultimate dividend date:

$$P = w^{\frac{r}{s}}\left[d_1 + d_2 w + \frac{cw^2\left(1-w^{n-1}\right)}{2\left(1-w\right)} + 100w^n\right] \qquad \text{for } n \geq 1$$

(2) For trades settling on or after the penultimate dividend date and where the trade occurs before the publication of the RPI that determines the redemption payment (see Annex B for some examples which illustrate which RPI determines the redemption payment):

$$P = w^{\frac{r}{s}}\left(d_1 + 100\right) \qquad \text{for } n = 0$$

In this case, it is possible to solve algebraically for yield in terms of price:

$$\rho = 2 \cdot \left[\left(\frac{d_1 + 100}{P}\right)^{\frac{s}{r}} - 1\right]$$

(3) Where the trade occurs after the publication of the RPI that determines the redemption payment, the index-linked gilt will effectively become a nominal (rather than a real) instrument and the formula for calculating the (real) dirty price from the <u>nominal</u> yield will be given by:

$$P = \left(\frac{1}{\text{Index Ratio}_{\text{Set Date}}}\right)\left[v^{\frac{r}{s}}\left(D_{LAST} + R\right)\right]$$

In this case, it is possible to solve algebraically for yield in terms of price:

$$y = 2 \cdot \left[\left(\frac{D_{LAST} + R}{P \times \text{Index Ratio}_{\text{Set Date}}}\right)^{\frac{s}{r}} - 1\right]$$

Where: P = Real dirty price per £100 nominal.

c = Coupon per £100 nominal.

r = Number of calendar days from the settlement date to the next quasi-coupon date ($r = s$ if the settlement date falls on a quasi-coupon date).

s = Number of calendar days in the full quasi-coupon period in which the settlement date occurs (i.e. between the prior quasi-coupon date and the following quasi-coupon date).

n = Number of full quasi-coupon periods from the next quasi-coupon date after the settlement date to the redemption date.

ρ = Semi-annually compounded real redemption yield (decimal), i.e. if the real yield is 2.5% then ρ = 0.025.

$$w = \frac{1}{1+\frac{\rho}{2}}$$

y = Semi-annually compounded nominal redemption yield (decimal), i.e. if the nominal yield is 5% then y = 0.05.

$$v = \frac{1}{1+\frac{y}{2}}$$

D_{LAST} = Final (fixed) coupon payment per £100 nominal of the gilt, as published.

R = Redemption payment (fixed) per £100 nominal of the gilt, as published.

and Index Ratio is defined in Annex B.

Double-dated Gilts

A double-dated gilt has a final maturity date and in addition an earlier maturity date, with HM Treasury having the right to redeem the gilt on any day between these two dates, provided that the relevant notice is given (usually 3 months). In order to calculate the redemption yield for such gilts it is necessary to make some assumption about when the gilt will be redeemed (where a specific redemption date has not yet been announced by the authorities).

Case 1: *The settlement date is at least x months before the first date in the redeemable band (where x is the period of notice required to be given to call the gilt as specified in its prospectus - usually 3 months).* Then the *yield / coupon rule* is used: if the nominal redemption yield y is greater than or equal to the coupon, the latest redemption date in the redeemable band is assumed; otherwise the earliest redemption date in the redeemable band is assumed. For price to yield calculations, the *par rule* is used: if the clean price (i.e. excluding accrued interest) is less than or equal to par, the latest redemption date in the redeemable band is assumed; otherwise the earliest redemption date in the redeemable band is assumed. Note that in certain boundary cases, the two rules above may not be equivalent.

Case 2: The settlement date is either less than x months before the first date in the redeemable band (where x is the period of notice required to be given to call the gilt as specified in its prospectus - usually 3 months), or the settlement date is in the redeemable band. Then if notice has not yet been given by the authorities that the gilt will be redeemed early, the latest redemption date in the redeemable band is assumed (irrespective of whether the nominal redemption yield y is greater than or less than the coupon, or whether the clean price is less than or greater than par).

Having made such an assumption about the redemption date, if this falls on a quasi-coupon date the formula for conventional gilts should be used; if it falls on a date which is not a quasi-coupon date, the following formula should be used:

$$P = v^{\frac{r}{s}} \left(d_1 + d_2 v + \frac{cv^2}{2(1-v)}(1-v^{n-1}) + (100 + d_f) \cdot v^n \cdot v^{\frac{t}{u}} \right) \qquad \text{for } n \geq 1$$

Where:

P = Dirty price per £100 nominal of the gilt[3].

d_1 = Cash flow due on next quasi-coupon date, per £100 nominal of the gilt (may be zero if the gilt has a long first dividend period or if the gilt settles in its ex-dividend period; or may be greater or less than $\frac{c}{2}$ during long or short first dividend periods respectively).

d_2 = Cash flow due on next but one quasi-coupon date, per £100 nominal of the gilt (may be greater than $\frac{c}{2}$ during long first dividend periods).

d_f = Partial coupon due on off-quasi-coupon redemption date, per £100 nominal of the gilt.

c = Coupon per £100 nominal of the gilt.

y = Semi-annually compounded nominal redemption yield (decimal), i.e. if the yield is 5% then $y = 0.05$.

v = $\frac{1}{1+\frac{y}{2}}$

r = Number of calendar days from the settlement date to the next quasi-coupon date ($r = s$ if the settlement date falls on a quasi-coupon date).

s = Number of calendar days in the full quasi-coupon period in which the settlement date occurs (i.e. between the prior quasi-coupon date and the following quasi-coupon date).

t = Number of calendar days from the redemption date to the preceding quasi-coupon date.

u = Number of calendar days in the full quasi-coupon period in which the redemption date occurs.

n = Number of full quasi-coupon periods from the next quasi-coupon date after the settlement date to the redemption date.

For $n = 0$, the equation reduces to:

(1) If the period between the settlement date and the redemption date spans a quasi-coupon date

$$P = v^{\frac{r}{s}}\left(d_1 + (100 + d_f)\cdot v^{\frac{t}{u}}\right)$$

(2) If the period between the settlement date and the redemption date does not span a quasi-coupon date

$$P = (100 + d_f)\cdot v^{\frac{t^*}{u}}$$

Where: t^* = Number of calendar days from the settlement date to the redemption date.

Undated Gilts

All current undated gilts in issue have a date after which they can be redeemed (for example, 3½% War Loan is dated '1952 or after'). In order to calculate the redemption yield for such gilts it is necessary to make some assumption about when the gilt will be redeemed (where a specific redemption date has not yet been announced by the authorities).

If notice has not yet been given by the authorities that the gilt will be redeemed early, it is assumed that the gilt will not be redeemed and the infinite cash flow formula should be used (see below), irrespective of whether the nominal redemption yield y is greater than or less than the coupon, or whether the clean price is less than or greater than par.

For an actual early redemption date, if this falls on a quasi-coupon date the formula for conventional gilts should be used; if it falls on a date which is not a quasi-coupon date, the following formula should be used:

$$P = v^{\frac{r}{s}}\left(d_1 + d_2 v + \frac{cv^2}{f(1-v)}\left(1-v^{n-1}\right) + \left(100 + d_f\right)\cdot v^n \cdot v^{\frac{t}{u}}\right) \text{ for } n \geq 1$$

Where:

P = Dirty price per £100 nominal of the gilt[3].

d_1 = Cash flow due on next quasi-coupon date, per £100 nominal of the gilt (may be zero if the gilt has a long first dividend period or if the gilt settles in its ex-dividend period; or may be greater or less than $\frac{c}{f}$ during long or short first dividend periods respectively).

d_2 = Cash flow due on next but one quasi-coupon date, per £100 nominal of the gilt (may be greater than $\frac{c}{f}$ during long first dividend periods).

d_f = Partial coupon due on off-quasi-coupon redemption date, per £100 nominal of the gilt.

c = Coupon per £100 nominal of the gilt.

y = Nominal redemption yield (decimal), i.e. if the yield is 5% then

$y = 0.05$.

f = Number of coupons payable on the gilt per year (f will be equal to 2 or 4).

v = $\frac{1}{1+\frac{y}{f}}$

r = Number of calendar days from the settlement date to the next quasi-coupon date ($r = s$ if the settlement date falls on a quasi-coupon date).

s = Number of calendar days in the full quasi-coupon period in which the settlement date occurs (i.e. between the prior quasi-coupon date and the following quasi-coupon date).

t = Number of calendar days from the redemption date to the preceding quasi-coupon date.

u = Number of calendar days in the full quasi-coupon period in which the redemption date occurs.

n = Number of full quasi-coupon periods from the next quasi-coupon date after the settlement date to the redemption date.

For $n = 0$, the equation reduces to:

(1) If the period between the settlement date and the redemption date spans a quasi-coupon date

$$P = v^{\frac{r}{s}}\left(d_1 + (100 + d_f)\cdot v^{\frac{t}{u}}\right)$$

(2) If the period between the settlement date and the redemption date does not span a quasi-coupon date

$$P = (100 + d_f)\cdot v^{\frac{t^*}{u}}$$

Where: t^* = Number of calendar days from the settlement date to the redemption date.

Infinite cash flow method: For an infinite set of cash flows (i.e. where it is assumed that the gilt will not be redeemed) we use the formula for a conventional gilt and take P to be the limit of the sum of the discounted cash flows as n (the number of full quasi-coupon periods from the next quasi-coupon date after the settlement date to the redemption date) tends to infinity. Since $|v|<1$, this limit exists and is equal to

$$P = v^{\frac{r}{s}}\left(d_1 + d_2 v + \frac{cv^2}{f(1-v)}\right)$$

Where:

- P = Dirty price per £100 nominal of the gilt[3].
- d_1 = Cash flow due on next quasi-coupon date, per £100 nominal of the gilt (may be zero if the gilt has a long first dividend period or if the gilt settles in its ex-dividend period; or may be greater or less than $\frac{c}{f}$ during long or short first dividend periods respectively).
- d_2 = Cash flow due on next but one quasi-coupon date, per £100 nominal of the gilt (may be greater than $\frac{c}{f}$ during long first dividend periods).
- c = Coupon per £100 nominal of the gilt.
- y = Nominal redemption yield (decimal), i.e. if the yield is 5% then $y = 0.05$.
- f = Number of coupons payable on the gilt per year (f will be equal to 2 or 4).
- v = $\frac{1}{1+\frac{y}{f}}$
- r = Number of calendar days from the settlement date to the next quasi-coupon date ($r = s$ if the settlement date falls on a quasi-coupon date).
- s = Number of calendar days in the full quasi-coupon period in which the settlement date occurs (i.e. between the prior quasi-coupon date and the following quasi-coupon date).

Strips

Certain conventional gilts are eligible to be ‘stripped’ into their separate cash flows, which are called ‘strips’. In February 1997 the Bank of England (as the UK government debt manager at that time) published a consultative paper seeking views on what standardised formula for computing market prices from gross redemption yields should be adopted to allow gilt strips to trade on a yield basis. The result of the consultation was indicated by Press Notices on 30 May 1997 and 12 June 1997. The market consensus was that the following method would be the most suitable for strips:

$$P = \frac{100}{\left(1+\frac{y}{2}\right)^{\frac{r}{s}+n}}$$

Where: P = Price per £100 nominal of the strip.

y = Strip gross redemption yield (decimal), i.e. if the yield is 5% then y = 0.05.

r = Number of calendar days from the settlement date to the next quasi-coupon date ($r = s$ if the settlement date falls on a quasi-coupon date).

s = Number of calendar days in the quasi-coupon period in which the settlement date occurs (i.e. between the prior quasi-coupon date and the following quasi-coupon date).

n = Number of full quasi-coupon periods from the next quasi-coupon date after the settlement date to the redemption date.

The settlement proceeds are rounded to the nearest penny on the traded nominal amount (with no intermediate rounding of price).

In this special case, we can solve algebraically for yield in terms of price:

$$y = 2 \cdot \left[\left(\frac{100}{P}\right)^{\frac{s}{r+ns}} - 1\right]$$

SECTION TWO: CALCULATION OF DIVIDEND PAYMENTS ON GILTS

New gilts are typically first issued part way through a quasi-coupon period. The DMO sets a non-standard first dividend on new gilts which are not issued on a quasi-coupon date: either 'short' if the period between the issue date and the payment date is shorter than a normal coupon period, or 'long' otherwise. The accrued interest paid by purchasers at the first issue is consequently zero. This section provides formulae for the calculation of non-standard first dividends, as well as dividends paid after the first payment.

(1) Standard dividend periods

(i) Conventional gilts, double-dated and undated gilts

$$\text{Dividend per £100 nominal} = \frac{c}{f}$$

Where: c = Coupon per £100 nominal of the gilt.

f = Number of coupons payable on the gilt per year (f will be equal to 2 or 4).

(ii) Index-linked gilts (8-month indexation lag)[4]

$$\text{Dividend per £100 nominal} = \frac{c}{2} \times \frac{RPID}{RPIB}$$

(See endnote 7 for how this should be rounded).

Where: c = Coupon per £100 nominal of the gilt.

$RPID$ = The RPI which fixes the next dividend payment for the gilt, i.e. the RPI scheduled to be published seven months prior to the month of the next dividend payment and relating to the month eight months prior to the month of the next dividend payment (for example, if the next dividend payment on the gilt will be in November then the RPI which fixes its value is the RPI for March of that year).

$RPIB$ = The Base RPI for the gilt, i.e. the RPI scheduled to be published seven months prior to the month of first issue of the gilt and relating to the month eight months prior to the month of first issue of the gilt (for example, if the gilt is first issued in November then its Base RPI is the RPI for March of that year).

(iii) Index-linked gilts (3-month indexation lag)[4]

$$\text{Dividend per £100 nominal} = \frac{c}{2} \times \text{Index Ratio}_{\text{Dividend Date}}$$

Where: c = Coupon per £100 nominal

and Index Ratio is defined in Annex B.

Coupon payments are rounded to the nearest 6th decimal place per £100 nominal.

(2) Short first dividend periods

(i) Conventional gilts

$$\text{Dividend per £100 nominal} = \frac{r}{s} \times \frac{c}{2}\text{, rounded to nearest 6}^{\text{th}}\text{ decimal place}$$

Where:
- c = Coupon per £100 nominal of the gilt.
- r = Number of calendar days from the issue date to the next (short) coupon date.
- s = Number of calendar days in the full quasi-coupon period in which the issue date occurs.

(ii) Index-linked gilts (3-month indexation lag)[4]

$$\text{Dividend per £100 nominal} = \frac{r}{s} \times \frac{c}{2} \times \text{Index Ratio}_{\text{Dividend Date}}$$

Where: c = Coupon per £100 nominal
r = Number of calendar days from the issue date to the next (short) coupon date.
s = Number of calendar days in the full quasi-coupon period in which the issue date occurs.

and Index Ratio is defined in Annex B.

Coupon payments are rounded to the nearest 6th decimal place per £100 nominal.

(3) Long first dividend periods

(i) Conventional gilts

$$\text{Dividend per £100 nominal} = \left(\frac{r}{s} + 1\right) \times \frac{c}{2}$$, rounded to nearest 6th decimal place

Where: c = Coupon per £100 nominal of the gilt.
r = Number of calendar days from the issue date to the next quasi-coupon date.
s = Number of calendar days in the full quasi-coupon period in which the issue date occurs.

(ii) Index-linked gilts (3-month indexation lag)[4]

$$\text{Dividend per £100 nominal} = \left(\frac{r}{s}+1\right)\times\frac{c}{2}\times\text{Index Ratio}_{\text{Dividend Date}}$$

Where: c = Coupon per £100 nominal

r = Number of calendar days from the issue date to the next quasi-coupon date.

s = Number of calendar days in the full quasi-coupon period in which the issue date occurs.

and Index Ratio is defined in Annex B.

Coupon payments are rounded to the nearest 6th decimal place per £100 nominal.

(4) Short final dividend periods for double-dated or undated gilts which have been called for redemption on a date which is not a quasi-coupon date

$$\text{Dividend per £100 nominal} = \frac{r}{s}\times\frac{c}{f}\text{, rounded to nearest 6th decimal place}$$

Where: c = Coupon per £100 nominal of the gilt.

f = Number of coupons payable on the gilt per year (f will be equal to 2 or 4).

r = Number of calendar days from the redemption date to the preceding quasi-coupon date.

s = Number of calendar days in the full quasi-coupon period in which the redemption date occurs.

SECTION THREE: CALCULATION OF ACCRUED INTEREST

While coupon payments on individual gilts are usually made only twice a year, gilts can be traded on any business day. Whenever a gilt changes hands on a day that is not a coupon payment date, the valuation of the gilt will reflect the proximity of the next coupon payment date. This is effected by the payment of accrued interest to compensate the seller for the period since the last coupon payment date during which the seller has held the gilt but for which he receives no coupon payment. The accrued interest for gilts is computed as follows[6] (based on the 'actual / actual' daycount convention effective from 1 November 1998):

The accrued interest on all gilts is rounded to the nearest penny on the traded nominal amount for calculating settlement proceeds.

(1) Standard dividend periods

(i) All gilts excluding index-linked gilts with a 3-month indexation lag

$$AI = \begin{cases} \frac{t}{s} \cdot d_1 & \text{if the settlement date occurs on or before the ex-dividend date} \\ \left(\frac{t}{s} - 1\right) \cdot d_1 & \text{if the settlement date occurs after the ex-dividend date} \end{cases}$$

Where:
- AI = Accrued interest per £100 nominal of the gilt.
- d_1 = Next dividend per £100 nominal of the gilt, as published.
- t = Number of calendar days from the previous quasi-coupon date to the settlement date (t = 0 if the settlement date falls on a quasi-coupon date).
- s = Number of calendar days in the full quasi-coupon period in which the settlement date occurs.

(ii) Index-linked gilts (3-month indexation lag)[4]

$$RAI = \begin{cases} \frac{t}{s} \times \frac{c}{2} & \text{if the settlement date occurs on or before the ex-dividend date} \\ \left(\frac{t}{s} - 1\right) \times \frac{c}{2} & \text{if the settlement date occurs after the ex-dividend date} \end{cases}$$

Where: *RAI* = Real accrued interest per £100 nominal of the gilt.

c = Coupon per £100 nominal of the gilt.

t = Number of calendar days from the previous quasi-coupon date to the settlement date ($t = 0$ if the settlement date falls on a quasi-coupon date).

s = Number of calendar days in the full quasi-coupon period in which the settlement date occurs.

Inflation-adjusted accrued interest = $RAI \times \text{Index Ratio}_{\text{Set Date}}$ (see Annex B for the calculation of $\text{Index Ratio}_{\text{Set Date}}$).

(2) Short first dividend periods

(i) Conventional gilts

$$AI = \begin{cases} \frac{t^*}{s} \times \frac{c}{2} & \text{if the settlement date occurs on or before the ex-dividend date} \\ \left(\frac{t^* - r}{s}\right) \times \frac{c}{2} & \text{if the settlement date occurs after the ex-dividend date} \end{cases}$$

Where: *AI* = Accrued interest per £100 nominal of the gilt.

c = Coupon per £100 nominal of the gilt.

*t** = Number of calendar days from the <u>issue</u> date to the settlement date.

s = Number of calendar days in the full quasi-coupon period in which the settlement date occurs.

r = Number of calendar days from the <u>issue</u> date to the next (short) coupon date.

(ii) Index-linked gilts (3-month indexation lag)[4]

$$RAI = \begin{cases} \dfrac{t^*}{s} \times \dfrac{c}{2} & \text{if the settlement date occurs on or before the ex-dividend date} \\ \left(\dfrac{t^* - r}{s}\right) \times \dfrac{c}{2} & \text{if the settlement date occurs after the ex-dividend date} \end{cases}$$

Where:

RAI = Real accrued interest per £100 nominal of the gilt.

c = Coupon per £100 nominal of the gilt.

t^* = Number of calendar days from the issue date to the settlement date.

s = Number of calendar days in the full quasi-coupon period in which the issue date occurs.

r = Number of calendar days from the issue date to the next (short) coupon date.

Inflation-adjusted accrued interest = $RAI \times \text{Index Ratio}_{\text{Set Date}}$ (see Annex B for the calculation of $\text{Index Ratio}_{\text{Set Date}}$).

(3) Long first dividend periods

(i) Conventional gilts

$$AI = \begin{cases} \dfrac{t^*}{s_1} \times \dfrac{c}{2} & \text{if the settlement date occurs during the first quasi-coupon period} \\ \left(\dfrac{r_1}{s_1} + \dfrac{r_2}{s_2}\right) \times \dfrac{c}{2} & \text{if the settlement date occurs during the second quasi-coupon period on or before the ex-dividend date} \\ \left(\dfrac{r_2}{s_2} - 1\right) \times \dfrac{c}{2} & \text{if the settlement date occurs during the second quasi-coupon period after the ex-dividend date} \end{cases}$$

Where:

AI = Accrued interest per £100 nominal of the gilt.

c = Coupon per £100 nominal of the gilt.

t^* = Number of calendar days from the issue date to the settlement date in the first quasi-coupon period (this term only applies if the gilt settles in the first quasi-coupon period).

r_1 = Number of calendar days from the issue date to the next quasi-coupon date.

r_2 = Number of calendar days from the quasi-coupon date after the issue date to the settlement date in the quasi-coupon period after the quasi-coupon period in which the issue date occurs (this term only applies if the gilt settles in the second quasi-coupon period).

s_1 = Number of calendar days in the full quasi-coupon period in which the issue date occurs.

s_2 = Number of calendar days in the full quasi-coupon period after the quasi-coupon period in which the issue date occurs.

(ii) Index-linked gilts (3-month indexation lag)[4]

$$RAI = \begin{cases} \frac{t^*}{s_1} \times \frac{c}{2} & \text{if the settlement date occurs during the first quasi - coupon period} \\ \left(\frac{r_1}{s_1} + \frac{r_2}{s_2}\right) \times \frac{c}{2} & \text{if the settlement date occurs during the second quasi - coupon period on or before the ex - dividend date} \\ \left(\frac{r_2}{s_2} - 1\right) \times \frac{c}{2} & \text{if the settlement date occurs during the second quasi - coupon period after the ex - dividend date} \end{cases}$$

Where: RAI = Real accrued interest per £100 nominal of the gilt.

c = Coupon per £100 nominal of the gilt.

t^* = Number of calendar days from the issue date to the settlement date in the first quasi-coupon period (this term only applies if the gilt settles in the first quasi-coupon period).

r_1 = Number of calendar days from the issue date to the next quasi-coupon date.

r_2 = Number of calendar days from the quasi-coupon date after the issue date to the settlement date in the quasi-coupon period after the quasi-coupon period in which the issue date occurs (this term only applies if the gilt settles in the second quasi-coupon period).

s_1 = Number of calendar days in the full quasi-coupon period in which the issue date occurs.

s_2 = Number of calendar days in the full quasi-coupon period after the quasi-coupon period in which the issue date occurs.

Inflation-adjusted accrued interest = $RAI \times \text{Index Ratio}_{\text{Set Date}}$ (see Annex B for the calculation of $\text{Index Ratio}_{\text{Set Date}}$).

(4) Short final dividend periods for double-dated or undated gilts which have been called for redemption on a date which is not a quasi-coupon date

$$AI = \frac{t^{**}}{s} \times \frac{c}{f} \qquad \text{if the settlement date occurs on or before the final ex-dividend date}$$

Where:

	AI	= Accrued interest per £100 nominal of the gilt.
	c	= Coupon per £100 nominal of the gilt.
	f	= Number of coupons payable on the gilt per year (f will be equal to 2 or 4).
	t^{**}	= Number of calendar days from the settlement date to the preceding quasi-coupon date.
	s	= Number of calendar days in the full quasi-coupon period in which the redemption date occurs.

ANNEX A: ESTIMATION OF THE NOMINAL VALUES OF FUTURE UNKNOWN CASH FLOWS ON INDEX-LINKED GILTS WITH AN 8-MONTH INDEXATION LAG

Embedded within the price / yield formula for index-linked gilts with an 8-month indexation lag, the nominal values of unknown future dividends are estimated as:

$$d_{i+1} = \frac{c}{2} \times \frac{a}{u^i} \quad 1 \le i \le n$$

Where:

d_{i+1} = Dividend due on (i+1)th quasi-coupon date after the settlement date, per £100 nominal of the gilt.

c = Coupon per £100 nominal of the gilt.

π = The assumed annual inflation rate (decimal) = 0.03.

u = $\left(\frac{1}{1+\pi}\right)^{\frac{1}{2}} = \left(\frac{1}{1.03}\right)^{\frac{1}{2}}$

$RPIB$ = The Base RPI for the gilt, i.e. the RPI scheduled to be published seven months prior to the month of first issue of the gilt and relating to the month eight months prior to the month of first issue of the gilt (for example, if the gilt is first issued in November then its Base RPI is the RPI for March of that year).

$RPIL$ = The latest published RPI at the time of valuation.

k = Number of months between the month of the RPI that defines the dividend due (or would ordinarily be due, in the case of a long first dividend or a gilt settling in its ex-dividend period) on the next quasi-coupon date and the month of the latest published RPI at the time of valuation. For example, if the RPI for January is the RPI that defines the dividend due (or would ordinarily be due, in the case of a long first dividend or a gilt settling in its ex-dividend period) on the next quasi-coupon date and the latest published RPI at the time of valuation is the RPI for April, then k = 3.

a = $\frac{RPIL}{RPIB} \cdot u^{\frac{2k}{12}}$

n = Number of full quasi-coupon periods from the next quasi-coupon date after the settlement date to the redemption date.

In addition, in most cases the RPI determining the redemption payment will not have been published, so that the nominal value of the redemption payment will not be known at the time of settlement. Embedded within the price / yield formula for index-linked gilts with an 8-month indexation lag, the nominal value of the redemption payment is estimated as:

$$R = 100 \times \frac{a}{u^n}$$

Where:

R = Redemption payment per £100 nominal of the gilt.

c = Coupon per £100 nominal of the gilt.

π = The assumed annual inflation rate (decimal) = 0.03.

$u = \left(\frac{1}{1+\pi}\right)^{\frac{1}{2}} = \left(\frac{1}{1.03}\right)^{\frac{1}{2}}$

$RPIB$ = The Base RPI for the gilt, i.e. the RPI scheduled to be published seven months prior to the month of first issue of the gilt and relating to the month eight months prior to the month of first issue of the gilt (for example, if the gilt is first issued in November then its Base RPI is the RPI for March of that year).

$RPIL$ = The latest published RPI at the time of valuation.

k = Number of months between the month of the RPI that defines the dividend due (or would ordinarily be due, in the case of a long first dividend or a gilt settling in its ex-dividend period) on the next quasi-coupon date and the month of the latest published RPI at the time of valuation. For example, if the RPI for January is the RPI that defines the dividend due (or would ordinarily be due, in the case of a long first dividend or a gilt settling in its ex-dividend period) on the next quasi-coupon date and the latest published RPI at the time of valuation is the RPI for April, then k = 3.

$a = \frac{RPIL}{RPIB} \cdot u^{\frac{2k}{12}}$

n = Number of full quasi-coupon periods from the next quasi-coupon date after the settlement date to the redemption date.

All estimated index-linked gilt cash flows are left unrounded for price / yield calculation purposes[7].

ANNEX B: METHOD OF INDEXATION FOR INDEX-LINKED GILTS WITH A 3-MONTH INDEXATION LAG

Index-linked gilts first issued from April 2005 employ the three-month lag indexation technique first used in the Canadian Real Return Bond (RRB) market, rather than the eight-month lag methodology previously used. In addition to using a shorter lag, RRB indexation is applied in a significantly different way to that for earlier index-linked gilt issues.

An Index Ratio is applied to calculate the coupon payments, the redemption payment (i.e. the uplifted principal) and the accrued interest. The Index Ratio for a given date is defined as the ratio of the reference RPI applicable to that date ($\text{Ref RPI}_{\text{Date}}$) divided by the reference RPI applicable to the original issue date of the gilt ($\text{Ref RPI}_{\text{First Issue Date}}$), rounded to the nearest 5th decimal place:

$$\text{Index Ratio}_{\text{Date}} = \left[\frac{\text{Ref RPI}_{\text{Date}}}{\text{Ref RPI}_{\text{First Issue Date}}}\right]\text{, rounded to the nearest 5}^{\text{th}}\text{ decimal place.}$$

The reference RPI for the first calendar day of any calendar month is the RPI for the calendar month falling three months earlier. For example, the reference RPI for 1 June corresponds to the RPI for March, the reference RPI for 1 July corresponds to the RPI for April, etc. The reference RPI for any other day in the month is calculated by linear interpolation between the reference RPI applicable to the first calendar day of the month in which the day falls and the reference RPI applicable to the first calendar day of the month immediately following. Interpolated values for $\text{Ref RPI}_{\text{Date}}$ should be rounded to the nearest 5th decimal place, as should values for $\text{Index Ratio}_{\text{Date}}$.

The formula used to calculate $\text{Ref RPI}_{\text{Date}}$ can be expressed as follows:

$$\text{Ref RPI}_{\text{Date}} = \text{Ref RPI}_{\text{M}} + \left(\frac{t-1}{D}\right)\left[\text{Ref RPI}_{\text{M+1}} - \text{Ref RPI}_{\text{M}}\right]$$

Where: D = Number of days in the calendar month in which the given date falls.

t = The calendar day corresponding to the given date.

$\text{Ref RPI}_{\text{M}}$ = Reference RPI for the first day of the calendar month in which the given date falls.

$\text{Ref RPI}_{\text{M+1}}$ = Reference RPI for the first day of the calendar month immediately following the given date.

For example, the reference RPI for 20 July 2001 is calculated as follows:

$$\text{Ref RPI}_{\text{20 July 2001}} = \text{Ref RPI}_{\text{1 July 2001}} + \left(\frac{19}{31}\right)\left[\text{Ref RPI}_{\text{1 August 2001}} - \text{Ref RPI}_{\text{1 July 2001}}\right]$$

$$= \text{RPI}_{\text{April 2001}} + \left(\frac{19}{31}\right)\left[\text{RPI}_{\text{May 2001}} - \text{RPI}_{\text{April 2001}}\right]$$

$$= 173.1 + \left(\frac{19}{31}\right)\left[174.2 - 173.1\right]$$ = 173.77419, when rounded to the nearest 5th decimal place.

The $\text{Ref RPI}_{\text{First Issue Date}}$ for a given bond remains constant over its life. However, different index-linked gilts should have different values for $\text{Ref RPI}_{\text{First Issue Date}}$ (depending on when they are first issued).

Calculation of the settlement price

Index-linked gilts with a three-month lag trade and are auctioned on the basis of the <u>real</u> clean price per £100 nominal.

The Inflation-adjusted clean price per £100 nominal is calculated from the real clean price using the following formula:

Inflation-adjusted clean price = $\text{Real clean price} \times \text{Index Ratio}_{\text{Set Date}}$ (this should be left unrounded).

The Inflation-adjusted dirty price per £100 nominal is calculated as:

Inflation-adjusted dirty price per £100 nominal = Inflation-adjusted clean price per £100 nominal + Inflation-adjusted accrued interest per £100 nominal (this should be left unrounded).

Where: Inflation-adjusted accrued interest = $\text{Real accrued interest} \times \text{Index Ratio}_{\text{Set Date}}$

and the Real accrued interest (*RAI*) is defined in Section 3. The Inflation-adjusted accrued interest should be left unrounded.

Calculation of the redemption payment

The redemption payment per £100 nominal is calculated as follows:

$$\text{Redemption Payment} = 100 \times \text{Index Ratio}_{\text{Redemption Date}}$$

The redemption payment is rounded to the nearest 6th decimal place per £100 nominal.

Note: unlike in some sovereign index-linked bond markets, in the UK no deflation floor will be applied when calculating the redemption payment, i.e. the redemption payment for an index-linked gilt could fall below £100 per £100 nominal if $\text{Ref RPI}_{\text{Redemption Date}}$ were less than $\text{Ref RPI}_{\text{First Issue Date}}$.

When does the redemption payment become known?

To illustrate when the redemption payment (and the final coupon payment) will be fixed, consider some hypothetical cases based on the assumption of an index-linked gilt with a three-month lag redeeming on different dates in December 2003.

Case 1: Redemption on 1 December 2003

The redemption payment would have been fixed when the September 2003 RPI was published on 14 October, i.e. the redemption payment would have been known approximately 6 weeks (48 days) before the bond redeemed.

Case 2: Redemption on 2 December 2003

The redemption payment would have been fixed when the October 2003 RPI was published on 18 November, i.e. the redemption payment would have been known 2 weeks (14 days) before the bond redeemed.

Case 3: Redemption on 31 December 2003

The redemption payment would have been fixed when the October 2003 RPI was published on 18 November, i.e. the redemption payment would have been known approximately 5 weeks (43 days) before the bond redeemed.

So, in practice, the redemption payment and the final dividend payment on an index-linked gilt with a three-month lag will typically be fixed around 2-6 weeks before the redemption date.

NOTES

1. In order to solve some types of equation it is necessary to obtain numerical approximations to the roots using an iterative process. An iterative process starts with an approximation x_0 to a root λ from which another approximation x_1 is obtained, and then another approximation x_2, and so on. For an effective process (for a particular root) the successive values (or iterates) $x_1, x_2, x_3, \ldots$ should become progressively closer to the root λ. The process is continued until an approximation of the required accuracy is obtained.

2. The section on double-dated and undated gilts provides more information on how to work out the assumed redemption date.

3. The dirty price of a gilt is its total settlement price which includes accrued interest.

4. The following conventions will apply in the (very rare) event that the Retail Prices Index is revised following an initial release.

	Index-linked gilts (8-month indexation lag)	**Index-linked gilts (3-month indexation lag)**
Dividends	Use first publication	Use first publication
Accrued interest	Use first publication	Use first publication
Price / yield	Use revised publication for *RPIL*	N/A (no RPI term)

where *RPIL* is the latest published RPI at the time of valuation.

5. If this has not yet been published by the authorities, see Annex A for how to estimate it.

6. The ex-dividend date for all gilts except 3½% War Loan is currently the date seven business days before the dividend date; for 3½% War Loan it is the date ten business days before the dividend date.

7. Actual (i.e. published) cash flows on index-linked gilts are rounded as follows per £100 nominal: (a) 2% IL 2006 and 2½% IL 2011: rounded down to 2 decimal places; (b) 2½% IL 2009, 2½% IL 2013, 2½% IL 2016, 2½% IL 2020, 2½% IL 2024

and 4⅛% IL 2030: rounded down to 4 decimal places; (c) all other index-linked gilts (i.e. those first issued after January 2002): rounded to the nearest 6th decimal place.